(Pages : 3) **N – 4037**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme Under CBCSS

Statistics

Core Course — I

ST 1141 — STATISTICAL METHODS

(2013-2017 Admissions)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Write the scope of Statistics.
2. Define coefficient of variation.
3. What do you mean by positional average?
4. Define mode of a data.
5. What do you mean by relative measure of dispersion?
6. Explain range of a data.
7. What is the limit of Spearman's correlation coefficient?
8. Define Skewness.
9. Write the regression equation for X on Y.
10. Define standard error.

(10 $\times$ 1 = 10 Marks)

P.T.O.

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Write the important step to create a questionnaire.

12. Explain Pie Chart.

13. Write the properties of geometric mean.

14. State the relationship between arithmetic mean, geometric mean and harmonic mean for non-negative observations.

15. Explain frequency distribution and interval frequency distribution.

16. Write any four absolute measure of dispersion.

17. Explain mean deviation about mean.

18. What are the properties of correlation coefficient?

19. Discuss about the positive and negative correlations.

20. Explain the correlation ratio.

21. Write the normal equations for fitting a exponential curve.

22. Define multiple correlation.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. Distinguish between primary and secondary data.

24. Explain the frequency curve and frequency polygon.

25. Find the arithmetic mean for the following, 1, 1/2, 1/4, 1/8, 1/16.

26. Explain the working steps of quartile deviation in interval frequency data.

27. Find the geometric mean for the following, $2, 2^2, 2^3, 2^4, 2^5$.

28. Let $V(X)$, be the variance of X. Then show that $V(2X-3)=4V(X)$.

29. Find Kurtosis for the following data, 85, 96, 76, 108, 85, 80, 100, 85, 70, 95.

30. Explain regression analysis.

31. Fit second order linear model for the following data.

X	12	13	16	18	20	21	22	23	24	25
Y	35	27	86	155	167	208	191	251	246	294

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. Find Mode for the following data :

Class	0-10	10-20	20-30	30-40	40-50	50-60
f	13	12	20	15	3	2

33. Let $21X+13Y=7$ and $7X+Y=1$ are two regression equations. Then find the means and correlation coefficients.

34. Find the Standard deviation and Mean deviation about mean for the following data :

Class	0-5	5-10	10-15	15-20	20-25	25-30	30-35
f	1	7	11	22	12	4	2

35. Fit a curve $Y=aX^b$ for the following data :

X	1	2	3	4	5	6	7	8	9	10
Y	10	9	8	7	6	5	4	3	2	1
y	33	33	36	42	44	39	33	39	1	1

(2 × 15 = 30 Marks)

(Pages : 3) **N – 4038**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme Under CBCSS

Statistics

Core Course I

ST 1141 : STATISTICAL METHODS I

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** question. Each carries **1** mark.

1. A histogram generally represents a for ______ type of data.
2. For any discrete distribution _________ will not be less than the Mean deviation from Mean.
3. Sum of absolute deviation is minimum when taken from ________
4. Representative part of the population is called __________
5. Sheppard's correction for second central moment is _________
6. What is the Median of 8,3,0,9,6.
7. State true or false "histogram is a one-dimensional diagram".
8. The algebraic sum of the deviation of observation of a set value from their arithmetic mean is ________
9. Find the Range of 20,10,5,8,2.
10. Define Harmonic mean.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer **any eight** questions. Each carries **2** marks.

11. Explain Skewness with the help of figure.
12. Explain Nominal scale and ordinal scale.
13. Find the numbers whose arithmetic mean is 12.5 and geometric mean 10.
14. Define two way classification. Give Example.
15. Write any four method for collecting primary data
16. List out any four properties for Arithmetic mean.
17. What are the merits of mode.
18. Compare Questionnaire and Schedule.
19. Define Deciles.
20. Explain pictogram.
21. Find Q3 and D9 for the data:282,754,125,765,875,645,985,235,175,895,905,112 and 155.
22. How statistics can be misused.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each carries **4** marks.

23. The mean of a group of 100 observations is known to be 50. Later it was discovered that two observations were misread as 92 and 8 instead of 192 and 88. Find the correct mean.
24. How will you compute Median for the frequency distribution.
25. Calculate Standard deviation for the following data

class	0-4	4-8	8-12	12-16	16-20
f	3	8	17	10	2

26. Calculate Geometric mean of 4,6,9,11 and 15.

27. For a certain distribution upper and lower quartiles are 56 and 44 respectively. If median for the same data is 55 then identify the nature of skewness.

28. What are the points to be remembered while taking secondary data?

29. Explain the construction of a pie chart.

30. Explain desirable properties of a good average.

31. Draw a percentage bar diagram for the following data.

Year	Computer	Arts	Law
2014	1100	1600	700
2015	1200	1405	800
2016	1050	1130	739

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each carries **15** marks.

32. Draw a less than frequency curve and greater than frequency curve and find median for the following data

Age	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No.of patients	7	10	21	27	22	9	4

33. (a) Write down the general guidelines helpful in drafting a questionnaire.

    (b) Explain the importance of Statistics in various sectors and disciplines.

34. Which is more consistent:

A	25	50	45	30	70
B	10	70	50	20	95

35. For the following data calculate coefficient of Skewness and coefficient of Kurtosis and Comment on it

Data	2	3	7	8	10

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 4039**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme Under CBCSS

Statistics

Core Course I

ST 1141 : STATISTICAL METHODS – I

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. What is the meaning of Statistics?

2. What are the limitations of Statistics?

3. Define a schedule.

4. What are cartograms?

5. What is tabulation?

6. What percentage of observations is between first and sixth decile?

7. Give the relation between range and S.D.

8. Name a positional average.

P.T.O.

9. Define raw moments of a distribution.

10. Kurtosis is adjudged around which measure of central tendency?

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. What are the misuses of Statistics?

12. Define primary data and secondary data.

13. What is a cumulative frequency table?

14. Draw a frequency polygon for the following data and hence find the mode :

Marks obtained :	<10	<20	<30	<40	<50
No. of students :	2	4	9	7	3

15. Distinguish between quantitative data and qualitative data.

16. Find the weighted A.M. of 2, 5, 9 and 11 with weights 8, 7, 3 and 2.

17. Calculate the G.M of 3, 6, 24 and 48.

18. If the variance of n consecutive natural numbers is 14, what is the value of n?

19. What is the relation between mean, median and mode for a moderately skewed distribution?

20. What are the demerits of mode?

21. Give the formula for the i^{th} percentile.

22. Give the relationships between Q.D, M.D and S.D.

23. For a symmetric distribution, how can one determine the upper and lower quartile with the help of Q.D and the median?

24. What is coefficient of variation? What is the implication of a large value of it?

25. What is the purpose of measuring averages, measures of dispersion, skewness and Kurtosis?

26. What do you mean by skewness?

(8×2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

27. What are the important methods of collecting a primary data?

28. What are the precautions to be taken in using a secondary data?

29. Define classification of data. What are the different types of classification?

30. What are the advantages and limitations of an average?

31. Find the H.M. of $1, \frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. What are its uses?

32. If G_1 is the G.M of n_1 observations and G_2 is the G.M of n_2 observations then find the G.M of the pooled set of $(n_1 + n_2)$ observations.

33. What are the requisites of a good measure of dispersion?

34. Differentiate between absolute and relative measures of dispersion.

35. Establish the relation between raw moments and central moments.

36. If the mean of a distribution is 15 and variance is 25, with the coefficient of skewness $\beta_1 = 1$, find the third raw moment.

37. Why Sheppard's correction is needed for the moments of grouped data. Give the Sheppard's correction for the first 4 central moments.

38. Discuss positive skewness of a distribution based on its characteristics.

(6×4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks.

39. (a) Discuss the important graphical representations of a data?

(b) The following data gives the age of a group of people. Draw the Histogram and hence find the median of the distribution.

Class interval :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency :	8	20	36	24	12

40. (a) Find the missing values from the following information :

	Group I	Group II	Group III	Combined
Number	50	?	90	200
S.D.	6	7	?	7.746
Mean	113	?	115	116

(b) Show that sum of deviations of observations from the mean is zero.

41. Find the mean and S.D from the following frequency distribution :

Heights in inches :	59 – 61	61 – 63	63 – 65	65 – 67	67 – 69
No. of students :	4	30	45	15	6

42. (a) Show that M.D is minimum when taken from the median.

(b) Calculate the mean deviation from the median of the following data.

X	10	11	12	13	14	Total
Frequency	3	12	18	12	3	48

43. (a) What are the important relative measures of dispersion?

(b) Find the Quartile deviation and the coefficient of Q.D from the data.

Wages :	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
No. of workers :	6	12	18	10	4

44. Compute the first four moments about mean (central moments) for the following data and compute the coefficients of skewness and kurtosis :

Classes :	10 – 12	12 – 14	14 – 16	16 – 18	18 – 20	20 – 22	22 – 24
Frequency :	1	3	7	20	12	4	3

(2 × 15 = 30 Marks)

(Pages : 4) **N – 4041**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Statistics

MM 1131.4 : MATHEMATICS I – BASIC CALCULUS FOR STATISTICS

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are compulsory. They carry **1** mark each.

1. Find the third derivative of the function $f(x) = x^3 \sin x$.

2. Define curvature of a curve.

3. Explain why there is no point *c* is the interval $(0, \pi)$ for $f(x) = \tan x$ such that $f'(c) = 0$, even though $f(0) = f(\pi) = 0$.

4. What is the sum of the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$?

5. Find the average value of $f(x) = 2x$ over $[0, 4]$.

6. Evaluate $\int_0^2 (2-x)^{-1/2}\, dx$.

7. State comparison test of convergence of series.

P.T.O.

8. Find the stationary points of $f(x) = 3x^5 - 5x^3$.

9. State Mean value theorem.

10. Write the n^{th} derivative of e^{2x}.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions from among the questions **11** to **22**. These questions carry **2** marks each.

11. Evaluate $\int x^3 e^{-x^2} dx$.

12. Given that Rolle's theorem holds with $b = 2 + 1/\sqrt{3}$ for the function $f(x) = x^3 - 6x^2 + ax + c$ on $(1, 3)$. Find the values of a and b.

13. Show that the maximum curvature of the catenary $y(x) = a\cosh(x/a)$ is $1/a$.

14. For the function $y(x) = x^2 \exp(-x)$, obtain a simple relationship between y and $\frac{dy}{dx}$.

15. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 1$.

16. Determine whether the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ converge absolutely.

17. Show that lowest value taken by the function $3x^4 + 4x^3 - 12x^2 + 6$ is –26.

18. Show that the curve $x^3 + y^3 - 8x - 12y - 16 = 0$ touches the y–axis.

19. Find the position and nature of the stationary points of the function $f(x) = \cos ax$ with a $\neq 0$.

20. Sum the series $S = 1 + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} + \ldots$

21. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+1)(n+2)}$.

22. Expand $f(x) = \cos x$ as a Taylor series about $x = \pi/2$.

(8 $\times$ 2 = 16 Marks)

SECTION – III

Answer any **six** questions from among the questions **23** to **31**. Each question carries **4** marks.

23. Use Leibnitz theorem to find the fourth derivative of $x^2 e^{3x}$.

24. Find the Maclaurin's series for $\ln\left(\frac{1+x}{1-x}\right)$.

25. Evaluate the integral $\int_0^2 (2-x)^{-1/4} dx$.

26. Determine the surface area of the cone generated by the line $y = 2x$ from $x = 0$ to $x = h$ about the x-axis.

27. Determine the range of values of x for which the power sides converges :

$P(x) = 1 + 2x + 4x^2 + 8x^3 + \ldots$.

28. What semi-quantities results can be deduced by applying Rolle's Theorem to the following functions $f(x)$, with a and c chosen so that $f(a) = f(c) = 0$?

(a) $x^2 - 6x + 8$

(b) $\sin x$.

29. State Cauchy's root test and use it to determine whether the following series converges $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n = 1 + \frac{1}{4} + \frac{1}{27} + \ldots$

30. Use Integration by parts to evaluate $\int_0^y x^2 \sin x \, dx$.

31. Evaluate the integral $\int \sin^5 x \, dx$.

(6 $\times$ 4 = 24 Marks)

SECTION – IV

Answer any **two** questions from among the questions **32** to **35**. These questions carry **15** marks.

32. (a) Show that the entire length of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ which can be parametrized as $x=a\cos^3\theta$ and $y=b\sin^3\theta$ is $6a$.

(b) Find the volume of the solid that is obtained when the region under the curve $y=\sqrt{x}$ over the interval $[1, 4]$ is revolved about the x–axis.

33. (a) Find the area of surface that is generated by revolving about the portion of the curve $y=x^3$ between $x=0$ and $x=1$ about the x-axis.

(b) Equation in polar coordinates of an ellipse with semi-axes a and b is

$\frac{1}{\rho^2}=\frac{\cos^2\phi}{a^2}+\frac{\sin^2\phi}{b^2}$. Find the area A of the ellipse.

34. (a) Discuss the convergence of Riemann Zeta series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ for $p>1$ and $p\le 1$.

(b) Using Lagrange's mean value theorem, prove that $\frac{c-a}{1+c^2}<\tan^{-1}c-\tan^{-1}a<\frac{c-a}{1+a^2}$.

35. (a) Determine the range of values of z for which the following complex power series converges :

$$P(z)=1-\frac{z}{2}+\frac{z^2}{4}-\frac{z^3}{8}+\dots$$

(b) Sum the series $S(\theta)=1+\cos\theta+\frac{\cos 2\theta}{2!}+\frac{\cos 3\theta}{3!}+\dots$

(2 × 15 = 30 Marks)

(Pages : 6) **N – 4042**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course I for Statistics

MM 1131.4 : MATHEMATICS I – BASIC CALCULUS FOR STATISTICS

(2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

(All the **first ten** questions are compulsory. They carry **1** mark each)

1. Find the third derivative of the function $f(x) = x^3 \sin x$.

2. Define curvature of a curve.

3. Explain why there is no point c in the interval $(0, \pi)$ for $f(x) = \tan x$ such that $f'(c) = 0$, even though $f(0) = f(\pi) = 0$.

4. What is the sum of the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$?

5. Find the average value of $f(x) = 2x$ over $[0,4]$.

6. Evaluate $\int_0^2 (2-x)^{-1/2} dx$.

7. State comparison test.

P.T.O.

8. Find the stationary points of $f(x)=3x^5-5x^3$.

9. State Mean value theorem

10. Write the n^{th} derivative of e^{2x}.

(10 $\times$ 1 = 10 Marks)

SECTION – II

(Answer **any eight** from among the questions 11 to 26. These questions carry **2** marks each)

11. Evaluate $\int x^3 e^{-x^2} dx$.

12. Evaluate the integral $\int_0^\infty \frac{x}{(x^2+a^2)^2} dx$.

13. Given that Rolle's Theorem holds with $b=2+1/\sqrt{3}$ for the function $f(x)=x^3-6x^2+ax+c$ on $(1,3)$. Find the values of *a* and *b.*

14. Find 'b'of the Mean Value Theorem when $f(x)=x(x-1)$ in $(1,2)$.

15. Show that the maximum curvature of the centenary $y(x)=a\cosh(x/a)$ is $1/a$.

16. For the function $y(x)=x^2\exp(-x)$, obtain a simple relationship between y and $\frac{dy}{dx}$.

17. Find the length of the curve $y=x^{3/2}$ from $x=0$ to $x=1$.

18. State the alternating series test.

19. Determine whether the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+....$ converge absolutely.

20. Show that lowest value taken by the function $3x^4 + 4x^3 - 12x^2 + 6$ is –26.

21. Show that the curve $x^3 + y^3 - 8x - 12y - 16 = 0$ touches the y-axis.

22. Find the position and nature of the stationary points of the function $f(x) = \cos ax$ with a $\neq 0$.

23. Sum the series $S = 1 + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} + ...$

24. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+1)(n+2)}$.

25. Expand $f(x) = \cos x$ as a Taylor series about $x = \pi/2$.

26. Find the inflexion points of $f(x) = x^4 - 2x^3 + 2x - 1$.

(8 $\times$ 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the question 27 t o38. These questions carry **4** marks.

27. Use Leibnitz theorem to find the fourth derivative of $x^2 e^{3x}$.

28. Determine inequalities satisfied by Inx and sin x for suitable ranges of the real variable x.

29. Find the Maclaurin's series for $n \left(\frac{1+x}{1-x}\right)$.

30. Evaluate the integral $\int_2^0 (2-x)^{-1/4} dx$.

31. Determine the surface area of the cone generated by the line $y = 2x$ from $x = 0$ to $x = 2$ about the x - axis.

32. Determine the range of values of x for which the power series converges:

$P(x) = 1 + 2x + 4x^2 + 8x^3 + ...$

33. What semi-quantities results can be deduced by applying Rolle's Theorem to the following functions $f(x)$, with a and c chosen so that $f(a)=f(c)=0$?

(a) x^2-6x+8

(b) $\sin x$

34. Given that the series $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, determine whether the following series converges $\sum_{n=1}^{\infty}\frac{4n^2-n-3}{n^3+2n}$.

35. State Cauchy's root test and use it to determine whether the following series converges:

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^n=1+\frac{1}{4}+\frac{1}{27}+...$$

36. Use integration by parts to evaluate $\int_0^y x^2\sin x\,dx$.

37. Evaluate the integral $\int\sin^5 x\,dx$.

38. Find the volume of a cone enclosed by the surface formed by rotating about the x-axis, the line $y=2x$ between $x=0$ and $x=h$.

(6 × 4 = 24 Marks)

SECTION – IV

(Answer **any two** questions from among the questions 39 to 44. These questions carry **15** marks each)

39. (a) Show that the entire length of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ which can be parametrized as $x=a\ \cos^3\theta$ and is 6a.

(b) Find the volume of the solid that is obtained when the region under the curve $y=\sqrt{x}$ over the interval $[1,4]$ is resolved about the x-axis.

40. (a) Find the area of surface that is generated by revolving about the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

(b) Equation in polar coordinates of an ellipse with semi-axes a and b is

$\frac{1}{\rho^2} = \frac{\cos^2\phi}{a^2} + \frac{\sin^2\phi}{b^2}$. Find the area A of the ellipse.

41. (a) Show that the radius of curvature at the point (x, y) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has magnitude $\frac{a^4y^2 + b^4x^2}{a^4b^4}$ and the opposite sign to y. Check the special case $b = a$, for which the ellipse becomes a circle.

(b) Show that the value of the integral $I = \int_0^1 \frac{1}{(1+x^2+x^3)^{1/2}}dx$ lies between 0.810 and 0.882.

42. (a) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for $p > 1$ and $p \le 1$.

(b) Using Lagrange's mean value theorm, prove that

$\frac{c-a}{1+c^2} < \tan^{-1}c - \tan^{-1}a < \frac{c-a}{1+a^2}$.

43. (a) Show that the series $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ converges.

(b) Sum the series $S(x) = \frac{x^4}{3(0!)} + \frac{x^5}{4(1!)} + \frac{x^6}{5(2!)} + ...$

44. (a) Determine the range of values of z for which the Following complex power series converges:

$$P(z) = 1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + ...$$

(b) Sum the series $S(\theta) = 1\cos\theta + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + ...$

(2 × 15 = 30 Marks)

(Pages : 6) **N – 4043**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Statistics

MM 1131.4 : MATHEMATICS I – DIFFERENTIAL CALCULUS

(2021 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** the questions.

1. Find $\lim_{x \to 0} \frac{|x|}{x}$.

2. Suppose that f and g are continuous functions such that $f(1) = 1$ and $\lim_{x \to 1} [f(x) + 3g(x)] = 10$, then find $g(1)$.

3. Find the derivative of $y = \sqrt{x^2 + 2}$.

4. Find the intervals on which $f(x) = x^3 - 4x + 3$ is decreasing.

5. Find the stationary points of the function $f(x) = x^3 - 3x + 2$.

6. Find $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cos x}{x - \frac{\pi}{2}}$.

7. Define Relative Minimum of a function.

8. Find the domain of the function $f(x,y) = \dfrac{xy}{x-2}$.

9. Find $\dfrac{\partial f}{\partial y}$ if $f(x,y) = \sin(xy)$.

10. Write the local linear approximation of a two variable function $f(x,y)$ at (x_0, y_0).

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions.

11. Find $\lim\limits_{x \to 4^-} \dfrac{x-2}{(4-x)(x+2)}$.

12. Find $\lim\limits_{x \to \infty} \dfrac{2(3x-4)}{3x+5}$.

13. Show that $|x-1|$ is continuous everywhere.

14. Find $\dfrac{dy}{dx}$ if $x^3 + 3xy = 15 + 2x^2 - y^3$.

15. Evaluate $I = \int \dfrac{e^x}{\sqrt{1-e^{2x}}} dx$.

16. Explain the first derivative test for checking extremum values of a function.

17. Evaluate $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{(1-\tan x)}{\cos 2x}$.

18. Define the concavity of a function.

19. Find $\dfrac{d^2y}{dx^2}$ if $x^3 - y^3 = 6$.

20. Show that the function $f(x)=\begin{cases}\dfrac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} & x \neq 0\\ 0 & x=0\end{cases}$ is not differentiable at $x=0$.

21. Find the level curves of a function $f(x,y)=x^2+4y^2$.

22. Find $\dfrac{\partial f}{\partial y}$ at $(1,1)$, if $f(x,y)=xe^{xy^2-1}$.

23. Find $\dfrac{\partial^2 f}{\partial x^2}$ at $(1,\pi)$, if $f(x,y)=x\cos(xy)$.

24. Let $f(x,y)=y^2x+5x^3$, then find the slope of the surface $z=f(x,y)$ at the point $(-1,1)$ in the y–direction.

25. Let $f(x,y)=x^2e^y+x$, find f_{yxx} at (1, 0).

26. If $r(x,y,z)=\sqrt{x^2+y^2+z^2}$ then find $\dfrac{\partial r}{\partial z}$.

(8 $\times$ 2 = 16 Marks)

SECTION – III

Answer any **six** questions.

27. Evaluate :

(a) $\lim\limits_{x\to\infty}\dfrac{\sqrt{5+4x^2}}{5+6x}$

(b) $\lim\limits_{x\to\infty}\sqrt{x^4+9}-x^2$.

28. Let $f(x)=\begin{cases}\dfrac{1}{x+2}; & x<-2\\ x^2-5; & -2<x\le 3\\ \sqrt{x+13}; & x>3\end{cases}$. Find the following limits

(a) $\lim_{x\to -2} f(x)$

(b) $\lim_{x\to 0} f(x)$

(c) $\lim_{x\to 3} f(x)$.

29. Find the positions and natures of the stationary points of the function $f(x)=2x^3-3x^2-36x+2$.

30. Show that the function $f(x)=\dfrac{1}{4}x^3+1$ satisfy the hypothesis of the mean value theorem over the interval [0, 2], and find all values of *c* in the interval (0, 2) at which the tangent line to the graph of *f* is parallel to the secant line joining the points $(0,f(0))$ and $(2,f(2))$.

31. Evaluate $\lim_{x\to 0}\left(1-\sin^2 x\right)^{\frac{1}{2x^2}}$.

32. Find $\dfrac{dy}{dx}$ if $y=\dfrac{(x-5)^2}{\left(x^2+1\right)}$.

33. Find the greatest and least values of $f(x)=3x^4-2x^3-6x^3+6x+1$ in [0, 2].

34. Identify the location of intercepts, relative extrema of the function $y=x^3-3x+2$.

35. If $f(x,y)=\begin{cases}-\dfrac{xy}{x^2+y^2} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$ then show that $f_x(x,y)$ and $f_y(x,y)$ exist at all points.

36. (a) Define local linear approximation $L(x,y)$ of $f(x,y)$ at (x_0,y_0).

(b) Find $L(x,y)$ at (3, 4) of $f(x,y)=\sqrt{x^2+y^2}$.

37. Suppose that $w=\sqrt{x^2+y^2+z^2}$, $x=\cos\theta, y=\sin\theta,, z=\tan\theta$. Use chain rule to find $\frac{dw}{d\theta}$ at $\theta=\frac{\pi}{4}$.

38. Explain Second derivative test. Find the local extreme value of the function $f(x,y)=xy-x^2-y^2-2x-2y+4$.

(6 $\times$ 4 = 24 Marks)

SECTION – IV

Answer any **two** questions.

39. (a) Use implicit differentiation to find $\frac{dy}{dx}$ for the Folium of Descartes $x^3+y^3=3xy$

(b) Find an equation for the tangent line to the Folium of Descartes at $\left(\frac{3}{2},\frac{3}{2}\right)$.

(c) At what points in the first quadrant is the tangent line to the Folium of Descartes horizontal.

40. (a) Find x such that

(i) $\log_{10} x=\sqrt{2}$

(ii) $5^x=7$

(iii) $\ln(x+1)=5$.

(b) A space shuttle taking off generates a sound level of 150 dB near the launch-pad. A person exposed to this level of sound would experience severe physical injury. By comparison, a car horn at one meter has a sound level of 110 dB, near the threshold of pain for many people. What is the ratio of sound intensity of a space shuttle take off to that of a car horn?

(c) Prove that $\frac{d}{dx}\ln(x)=\frac{1}{x}$.

41. (a) State mean value theorem Determine all the numbers ***c*** which satisfy the conclusion of mean value theorem for the function $f(x) = x^3 + 2x^2 - x$ on $[-1, 2]$.

(b) Verify Roll's theorem for the function $f(x) = x^2 + 2x - 8$ in [–4, 2].

42. (a) Evaluate $\lim_{x \to 0} (\cot x)^{1/\log x}$

(b) Calculate $\lim_{x \to 0^+} \left[\frac{1}{x} - \frac{1}{\sin x} \right]$.

(c) Find $\lim_{x \to 0} [1 + \sin x]^{\frac{1}{x}}$.

43. (a) (i) If $f(x, y) = xy + x^2 - 4$ find

(ii) $f(x + y, x - y)$ and

(iii) $f(xy, 3x^2y^3)$.

(b) If $f(x, y) = xy^2 + \cos(xy)$ then find all second order partial derivatives of $f(x, y)$

(c) $f(x, y, z) = 2x^2 + 3y^2 + 3z^2$ then find $f_{xx}(x, y) + f_{yy}(x, y) + f_{zz}(x, y)$ at (1, 1, 1).

44. (a) Find the point *p(x, y, z)* closest to origin on the plane $2x + y - z = 0$.

(b) Find the greatest and smallest values of the function $f(x, y) = xy$ takes on ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

(2 × 15 = 30 Marks)

(Pages : 4) **N – 4045**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Physics

Complementary Course for Statistics

PY 1131.3 — MECHANICS AND PROPERTIES OF MATTER

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions in **one** or **two** sentences; each question carries **1** mark.

1. Why does moment of inertia not change?

2. What is the function of a fly wheel?

3. What do you mean by oscillatory motion? Giver two examples.

4. What are the characteristics of simple harmonic motion?

5. Give an example for transverse and longitudinal waves?

6. Define a rigid body.

7. Why molecules on the surface of liquid have more energy?

8. What is meant by streamline flow of a liquid?

P.T.O.

9. Define coefficient of viscosity?

10. Explain a beam. **(10 $\times$ 1 = 10 Marks)**

SECTION – B

Answer **any eight** questions, not exceeding a paragraph; each question carries **2** marks.

11. Obtain an expression for moment of intertia of a uniform rigid rod?

12. State and prove perpendicular axis theorem?

13. How is longitudinal wave formed in air? Why transverse waves are not produced in gases?

14. What is torsional oscillation? What is the torsion constant of this pendulum?

15. What is the difference between free oscillations and forced oscillations?

16. What is a cantilever? Obtain an expression for the depression at the loaded end of a cantilever?

17. Define the angle of twist and angle of shear?

18. What is flexural rigidity and its expression?

19. Why does surface tension occur?

20. How surface tension is related to surface energy? What causes surface energy?

21. How does soluble impurities affect surface tension?

22. How does viscosity change with temperature?

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six,** each questions carries **4** marks.

23. Calculate the moment of inertia of a disc of mass 1.2 Kg and radius 8cm about

    (a) its diameter

    (b) an axis parallel to a diameter and tangential to the disc.

24. What is the angular momentum of a particle whose rotational kinetic energy is 18 joules, if the angular momentum vector coincides with the axis of rotation and its moment of inertia about the axis is 0.01 kgm^2.

25. A particle moving with simple harmonic motion has a period 0.001 s and amplitude 0.5cm. Find the acceleration, when it is 0.2cm apart from its mean position and its maximum velocity?

26. A sphere of mass 0.8 kg and radius 0.03 m is suspended from a wire of length 1 m and radius 5×10^4 m. If the period of torsional oscillations of this system is 1.23 sec. Calculate the modulus of rigidity of the wire.

27. Calculate the twisting couple on a solid shaft of length 1.5 m and diameter 120 mm when **it** is twisted through an angle 0.6°. The coefficient of rigidity for the material of the shaft may be taken to be 93×10^9 N/m^2.

28. A soap bubble is spherical in shape and has a diameter of 10 cm. If the surface tension of the surface separating soap solution and air is 40×10^{-3} N/m. What is the excess pressure of the air in the bubble over the atmospheric pressure?

29. A capillary tube 10^{-3} m in diameter and 0.2m in length is fitted horizontally to a vessel kept full of alcohol of density 0.8×10^{-3} kg/m^3. The capillary tube is 0.3m below from the surface of the alcohol in the vessel. Calculate the volume of the alcohol flows in 5 minute.

30. An air bubble of radius 1 cm is allowed to rise through a long cylindrical column of viscous liquid and travel at a steady rate of 0.21 cm/s. If the density of the liquid is 1470 kg/m^3, find the viscosity of the liquid. Neglect the density of the air.

31. Calculate the work done against surface tension force in blowing a soap bubble of *5* cm radius if the surface tension of soap solution is 0.025n/m

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions; each question carries **15** marks.

32. Derive an expression for the energy density of the plane progressive waves.

33. Deduce an expression for the twisting couple per unit twist of a uniform solid cylinder?

34. Discuss Jaeger's method for determining the surface tension of given liquid?

35. Obtain an expression for the moment of inertia of a fly wheel?

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 4046**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme Under CBCSS

Physics

Complementary Course for Statistics

PY 1131.3 –– MECHANICS AND PROPERTIES OF MATTER

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions in **one** or **two** sentences. **Each** question carries **1** mark.

1. Define rigidity modulus.

2. State the theorem of parallel axes.

3. What is meant by force of cohesion?

4. What is the moment of inertia of a circular disc about an axis through its center and perpendicular to its plane?

5. Define simple harmonic motion.

6. What is viscous force?

7. What are the two forces that govern the shape of a liquid drop?

8. Define the intensity of a wave.

P.T.O.

9. What is a beam?

10. Give the general equation for wave motion.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions, not exceeding a paragraph. **Each** question carries **2** marks.

11. Derive the expression for excess pressure inside a spherical drop.

12. What are I shaped girders used in the construction of bridges and railway tracks?

13. If the frequency of a simple harmonic oscillator is doubled, what will be the change in its total energy?

14. Explain why small liquid drops are spherical in shape.

15. Differentiate between angle of twist and angle of shear.

16. The small space between two parallel plates is filled with water. Why is it easier to separate the plates by sliding one over the other than by a direct pull?

17. Explain the variation of viscosity with temperature.

18. Show that potential energy per unit volume of stretched wire is U = 1/2(stress) $\times$ (strain).

19. What is the relationship between the kinetic energy of a rotating body and its angular velocity?

20. Why is a cantilever of uniform cross section more likely to break near its fixed end?

21. Write down the theory behind Ostwald's viscometer.

22. State two differences between translatory motion and rotatory motion.

23. What is elastic after effect?

24. What is radius of gyration?

25. Define : (i) neutral axis (ii) bending moment of a beam.

26. Explain the physical significance of moment of inertia.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

27. A circular disc of radius 0.1 m and mass 1 kg is rotating at a the rate of 10 revolutions per second about an axis at right angles to its plane and passing through its centre. Calculate the moment of inertia of the disc.

28. Two plane pieces of glass have water between them which is circular and of diameter 12cm. If the glass plates 0.6mm apart, what force perpendicular to the plates will be needed to separate them? (surface tension of water = 0.072 N/m)

29. Calculate the volume of water flowing in 10 minutes through a tube of 0.1 cm diameter, 40 cm long, if there is a constant pressure of $1.96 \times 10^3 N/m^2$. (Coefficient of viscosity of water = 0.89×10^{-3} Nsm^{-2})

30. Plane harmonic waves of frequency 500 Hz are produced in air with displacement amplitude of 10^{-5} m. Find the energy density and energy flux in the wave. (Density =1.29 kg/m^3 ,velocity = 340 m/s)

31. A bar of length 60 cm, breadth 3cm and thickness 4 mm is used as a cantilever. When a load of 0.25 kg is attached to the free end the depression at the free is 1 cm .Calculate the Young's Modulus of the material.

32. A wire 2 m long and 10^{-3} m diameter is fixed at one end. Find the couple required to twist the other end through 90°. (Rigidity modulus = 2.8×10^{10} N/m^2)

33. What is the radius of one large spherical drop formed when 1000 droplets of water each 10^{-6} cm in diameter coalesce?

34. The equation of a simple harmonic oscillator is given by $d^2x/dt^2 + 144x = 0$ Find the time period and frequency of oscillation.

35. What is the excess pressure of air in a spherical soap bubble of diameter 0.1 metre? (Surface Tension = 0.04 N/m)

36. What is the percentage increase in the time period of a simple pendulum, if its length is increased by 44%?

37. Consider a disc of mass 0.1 kg and radius 5 cm. Calculate the radius of gyration of this disc about an axis passing through its centre of gravity and perpendicular to its plane.

38. What is the wavelength of longitudinal waves of frequency 400 Hz in an alloy whose density is 5500 kg/m^3 and Young's Modulus 8.8×10^{10} N/m^2?

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. Write down the expression for a plane progressive wave explaining its symbols. Derive the expression for the velocity of transverse waves in a stretched string.

40. What is a rigid body? Derive the expression for the moment of inertia of a solid sphere.

41. Derive an expression for the depression produced at the free end of cantilever loaded at its free end.

42. What is a flywheel? Explain in detail the theory and experiment for the determination of moment of inertia of flywheel.

43. What is coefficient of viscosity? Derive Poiseulle's formula for the rate of flow of a liquid through a capillary tube.

44. Define surface tension and angle of contact. Explain how the surface tension of a liquid determined by measuring excess pressure?

(2 $\times$ 15 = 30 Marks)

(Pages : 3) **N – 4049**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme Under CBCSS

Computer Science

Complementary Course for Mathematics and Statistics

CS 1131.2/CS 1131.3 : INTRODUCTION TO IT

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A (Very Short Answer Type)

(One word to maximum of **one** sentence. Answer **all** questions).

1. POST stands for ____________

2. HTML is the abbreviation of ____________

3. Name 2 Web browsers.

4. ASCII stands for ____________

5. Expand RDRAM.

6. EEPROM stands for ____________

7. Give examples for 2 search engines.

8. GUI stands for ____________

P.T.O.

9. WLL stands for ___________

10. Expand URL

(10 $\times$ 1 = 10 Marks)

SECTION – B (Short Answer)

(Not to exceed **one** paragraph answer **any eight** questions. each question carries **2** marks)

11. What is auxiliary memory? Give an example.

12. What is SDRAM?

13. What are digital computers?

14. What is system software?

15. What are spreadsheets? Give an example.

16. Differentiate compiler and interpreter.

17. What is a Web server?

18. What is ISP?

19. What is the purpose of a joystick?

20. What do you mean by a leased line?

21. What is a router?

22. What is flash memory?

23. What is the purpose of a speaker?

24. What is a cable modem?

25. What is a workstation?

26. What is a word processor? Give example.

(8 $\times$ 2 = 16 Marks)

SECTION – C (Short Essay)

(Not to exceed **120** words, answer **any six,** questions. Each question carries **4** mark).

27. Write short notes on plotters.
28. Write short notes on memory stick.
29. Explain the use of sound cards.
30. What are presentation softwares?
31. Define computer virus. How will you protect computers from viruses?
32. Explain dial-up internet connection and DSL.
33. Write short notes on databases
34. Explain the concept of files and folders.
35. Write short notes on LaTex software.
36. Write short notes on free softwares.
37. Write notes on floppy disk.
38. Differentiate OMR and OCR.

(6 $\times$ 4 = 24 Marks)

SECTION – D (Long Essay)

(Answer **any two** questions, each question carries **15** mark).

39. Explain various generation of computers.
40. With a diagram explain the functional units of a computer.
41. Explain the working of CRT with a diagram.
42. Explain different types of operating system.
43. Discuss various types of printers.
44. Explain the characteristics of e-mail software.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **M – 2399**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Mathematics

Complementary Course for Statistics

MM 1231.4 : MATHEMATICS II – DIFFERENTIAL CALCULUS

(2014 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory.

They carry **1** mark each.

1. State triangle inequality.

2. Find $\lim_{x\to 2} \frac{x^2-4}{x-2}$.

3. If $f(x) = x^3 - x$, what is $f'(x)$?

4. What is rectilinear motion?

5. What will be the n^{th} term of the series $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, ...$?

6. Define sequence.

P.T.O.

7. Find the limit of the sequence $\left\{\dfrac{n}{2n+1}\right\}_{n=1}^{+\infty}$.

8. If $f(x, y) = 3x^2\sqrt{y} - 1$, find $f(0, 9)$.

9. Find $\lim\limits_{(x, y)\to(1, 4)} (5x^3y^2 - 9)$.

10. Find $\dfrac{\partial z}{\partial y}$, $z = x^4 \sin(xy^3)$.

SECTION – II

Answer **any eight** questions from among the questions 11 to 22.

These questions carry **2** marks each.

11. Find domain and range $f(x) = 2 + \sqrt{x-1}$.

12. Find $(f \circ g)$ and $(g \circ f)$ where $f(x) = x^2 + 3$ and $g(x) = \sqrt{x}$.

13. Let $s(t) = t^3 - 6t^2$ be the position function of a particle moving along an s-axis, where s is in meters and t is in seconds. Find the instantaneous acceleration.

14. State and prove constant difference theorem.

15. Find the limit of the sequence $\left\{\dfrac{n}{e^n}\right\}_{n=1}^{\infty}$

16. If $\lim\limits_{n\to\infty} |a_n| = 0$, then prove that $\lim\limits_{n\to\infty} a_{n=0}$.

17. Find the Taylor series for $\dfrac{1}{x}$.

18. Find the derivative of e^x using power series expansion.

19. Let $f(x, y) = \dfrac{-xy}{x^2 + y^2}$. Find the limit of $f(x,y)$ as (x,y) tends to (0,0) along

(a) the y-axis

(b) the parabola $y = x^2$.

20. Find the slope of the surface $f(x, y) = x^2 y + 5y^3$ in the y-direction at the point (1, –2).

21. Show that $u(x, t) = \sin(x - ct)$ is a solution of $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$.

22. Find the critical points of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$.

SECTION – III

Answer **any six** questions from among the questions 23 to 31.

These questions carry **4** marks each.

23. If $f(x) = 1 + \sqrt{x - 2}$ and $g(x) = x - 3$, find $f + g$, $f - g$, fg and f/g.

24. Sketch the graph of $y = \sqrt{x + 3}$.

25. Find $\lim\limits_{x \to \infty} \sqrt{x^6 + 5} - x^3$.

26. Find the n^{th} Maclaurin polynomial for $\dfrac{1}{1 - x}$.

27. Use numerical evidence to make a conjecture about limit of the sequence $\left\{\dfrac{n!}{n^n}\right\}_{n=1}^{+\infty}$ and then confirm that the conjecture is true.

28. Use Macluarin series to evaluate an approximate value of e.

29. Determine the level surfaces of $z^2 - x^2 - y^2 = k$.

30. Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in the y-direction at the point $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

31. Find all second order partial derivatives of $f(x, y) = x^2y^3 + x^4y$.

SECTION – IV

Answer **any two** questions from among the questions 32 to 35.

These questions carry **15** marks each.

32. Prove that

(a) $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$

(b) $\frac{d}{dx}(f(x) + g(x)) = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$.

(c) Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 cm and height 10 cm.

33. Find a point on the curve $y = x^2$ that is closest to the point (18,0).

34. (a) Find the interval of convergence and radius of convergence of $\sum_{k=0}^{\infty} k!x^k$.

(b) Find an approximate value of the integral $\int_0^1 e^{-x^2}dx$.

35. At what point or points on the circle $x^2 + y^2 = 1$ does the function $f(x,y) = xy$ have absolute maximum and what is the maximum value?

(Pages : 3) **M – 2400**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Physics

Complementary Course for Statistics

PY 1231.3 : THERMAL PHYSICS AND STATISTICAL MECHANICS

(2014 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

PART – A (Very short answer type)

Answer **all** questions in **one** word or maximum of **two** sentences. Each question carries **1** mark.

1. State Kelvin statement of the second law of thermodynamics.
2. Write down Rayleigh-Jeans law.
3. What is an adiabatic process?
4. What is adiabatic elasticity?
5. Write the expression for isothermal elasticity.
6. Write down the expression for work done by an ideal gas in an isothermal process.

P.T.O.

7. Define entropy.

8. Is a proton a Fermion or a Boson?

9. State Wiedmann-Franz law.

10. Define thermal conductivity.

($10 \times 1 = 10$ Marks)

PART – B (Short answer)

Answer **any eight** questions in about **one** paragraph. Each question carries **2** marks.

11. Distinguish between reversible and irreversible processes.

12. Explain Stefan's law.

13. State and explain Carnot's theorem.

14. Describe the planck radiation formula.

15. Explain fermi energy.

16. Describe the Einstein model of specific heat of a solid.

17. Discuss the change in entropy during a reversible process.

18. Compare average velocity, root mean square velocity and most probable velocity.

19. Derive the relation $C_p - C_v = nR$.

20. Describe the connection between entropy and available energy.

21. Explain ensemble.

22. What is meant by phase space?

($8 \times 2 = 16$ Marks)

PART – C (Short Essay)

Answer **any six** questions. Each carries **4** marks.

23. A Carnot engine of source temperature 500 K absorbs 300 joule of heat and rejects 150 calorie into the sink. Find the temperature of the sink and the efficiency.

24. Find the work done by 5 moles of hydrogen gas when it expands to thrice its initial volume at constant temperature of 400 K.

25. An automobile tyre has a pressure of 3 atoms at room temperature of 27°C. If it suddenly bursts, find its resulting temperature. Take $\gamma = 1.4$.

26. Explain the Carnot cycle with suitable diagram.

27. Explain the change in entropy when ice is converted into steam.

28. State and explain Wein's displacement law.

29. Compare Maxwell-Boltzmann, Fermi-Dirac and Bose-Einstein statistics.

30. Derive the expression for work done by an ideal gas during an adiabatic process.

31. How does the Rayleigh-Jeans law fail to explain the black body spectrum?

(6 $\times$ 4 = 24 Marks)

PART – D (Long Essay)

Answer **any two** questions. Each carries **15** marks.

32. Describe the Lee's disc experiment to measure thermal conductivity.

33. Explain the specific heat of electrons in a metal using Fermi-Dirac statistics.

34. Derive the expression for molecular energy distribution of an ideal gas.

35. Describe the working of a Carnot engine and hence derive the expression for its efficiency.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **M – 2402**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Computer Science

Complementary Course for Mathematics/Statistics

CS 1231.3/CS 1231.2 : PROGRAMMING IN C

(2016 – 2019 Admission)

Time : 3 Hours Max. Marks : 80

PART – A (Very Short Answer)

(One word to maximum of **one** sentence. Answer all questions. Each question carries **1** mark.

1. What is a STRING?
2. What is an array?
3. Give an example for derived data type.
4. What is a preprocessor directive?
5. What is a character constant?
6. What is a delimiter?
7. Explain the continue statement.
8. What is a function?

P.T.O.

9. What is an alternative for if else ladder?

10. What is meant by dynamic memory allocation?

(10 $\times$ 1 = 10 Marks)

PART – B (Short answer)

(Not to exceed **one** paragraph, answer **any eight** questions. Each question carries **2** marks)

11. How a pointer variable differs front an ordinary variable?

12. What is meant by debugging?

13. Write the output of the following program

```
#include<stdio.h>

void main ( )

{

  int x, y;

  x=y=10;

  printf (“%d\t%d”, ++x, y++);

}
```

14. How can you declare a constant in a C program?

15. Explain the logical operators in C.

16. What is meant by language translator?

17. Differentiate between local and global variable.

18. How can you return a value from a function?

19. Explain the conditional operator.

20. Explain the data type conversion with an example.

21. What is a self referencing structure?

22. What is a recursive function?

($8 \times 2 = 16$ Marks)

PART – C (Short Essay)

(Not to exceed **120** words, answer **any six** questions. Each question carries **4** marks).

23. Explain the general structure of a C program. Discuss the significance of each section.

24. Explain the unconditional branching and jump statements with example.

25. What is a variable? Explain the rule for giving variable name.

26. What are the storage classes n a C program?

27. Explain the fundamental data types in C language.

28. Differentiate between precedence and associativity of operators.

29. Explain the increment and decrement operators. What are its different forms?

30. Write a C program to check whether a string given as input is palindrome or not.

31. Debug the following code segment

```
int x y
scanf ("%d\t%d", x, y);
If (x > y)
y = x+++;
```

($6 \times 4 = 24$ Marks)

PART – D (Long Essay)

Answer **any two** questions. Each question carries **15** marks).

32. Write a program to read 10 numbers into an array and perform following operations

    (a) sort the numbers in descending order and display sorted list

    (b) Find the biggest and smallest.

33. Explain the loop statements along with suitable examples.

34. What is meant by user defined function? Explain the steps involved in using a user defined function. Discuss various methods for calling a function.

35. What is a flow chart? Explain the symbols used in drawing a flow chart. Draw a flow chart to check whether a number is prime or composite.

($2 \times 15 = 30$ Marks)

(Pages : 3) **M – 2403**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Statistics

Foundation Course – II

ST 1221 — STATISTICAL METHODS – II

(2018 – 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions each carry **1** mark.

1. Define correlation.
2. What is meant by perfect correlation?
3. What are the demerits of scatter diagram?
4. What are the merits of Rank Correlation?
5. Define data mining.
6. Where will the regression line meet?
7. State two properties of Regression.
8. When the regressions lines are parallel?
9. What is the output of R command rep (1, 4)?
10. What do you mean by data warehousing?

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer **any eight** questions. **Each** carries **2** marks.

11. What are the properties of regression coefficients?
12. What do you mean by coefficient of determination?
13. What do you mean by Link Analysis?
14. Describe R as a statistical software.
15. How to enter a data in R console?
16. How do you install R in your computer?
17. What do you mean by work space? Explain how to save the work space.
18. What is partial and multiple correlation?
19. What is a Decision tree?
20. How do you fit an exponential curve by the method of least squares?
21. What is predictive data mining?
22. What is Logistic Regression?

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions each carries **4** marks.

23. Define Correlation ratio. When Correlation ratio a more suitable measure than the correlation coefficient?
24. Explain what are regression lines. Why are there two regression lines?
25. Derive the expression for the angle between two regression lines.
26. Explain some methods of data input in R.
27. Write the build in functions for :
    (a) Finding Mode
    (b) Sorting a raw data in ascending and descending order.
28. Write down important functions in excel.

29. Fit a curve of the form $Y = ab^X$

X	40	65	90	5	30	10	80	88	70	25
Y	30	20	10	80	40	65	15	15	20	50

30. Derive the expression for the rank correlation coefficient.

31. (a) Explain classification in data mining.

    (b) Give the usage of time series in data mining.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions each carries **15** marks.

32. Following data gives the marks in 2 subjects in an examination. Mean mark in A = 52, Mean mark in B = 48, Standard deviation of marks in A = 15, Standard deviation of marks in B = 13, Correlation coefficient between marks in A and marks in B is 0.6

    (a) Draw the two lines of Regression

    (b) Give the estimate of marks in B for candidate who secured 50 marks in A.

    (c) How you identify the regression lines?

33. (a) Prove that the correlation coefficient lies between – 1 and +1

    (b) Calculate the correlation coefficient for the following data

X	5	6	7	9	12	15	14	16
Y	10	15	16	20	22	25	24	26

34. (a) Explain the arguments of a plot( ) function.

    (b) Explain how will you import data in R from excel?

    (c) Write a short note on data accessing and indexing.

35. (a) What is artificial neural network?

    (b) Explain different datamining tools.

    (c) Discriminant analysis.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **M – 2404**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Statistics

MM 1231.4 — MATHEMATICS – II – ADVANCED DIFFERENTIAL AND INTEGRAL CALCULUS

(2018 – 2019 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each question carries **1** mark.

1. Check whether the differential $x^2 dy + 2xy\, dx$ is exact.
2. Give the Maclurian's series expansion of a function $f(x, y)$ of two variables x and y about the origin.
3. Write condition for which a point (x_1, n_1) to be a critical point of the function $f(x, y)$.
4. Find the value of the double integral $\int_{-1}^{1}\int_{0}^{2} x^3 dy\; dx$.
5. Find the Jacobian of the transformation $x = r\cos\theta$, $y = r\sin\theta$ with respect to r and θ.
6. Find the value of the tripple integral $\int_{0}^{2}\int_{-1}^{1}\int_{0}^{3} x\, y^2 z dx\, dy\, dz$.

P.T.O.

7. The value of $\Gamma(3)$ is.

8. The value of $\beta(m,n)\Gamma(m+n)-\Gamma(m)\Gamma(n)$ is.

9. Write the value of $\Gamma\left(\frac{1}{2}\right)$.

10. Write the value of the factorial $n!$ as an integral.

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. If $f(t)=t$ and $y(t)=1+t$. Find the rate of change of $f(x,y)=xy$ with respect to 't'.

12. Find the total derivative of $f(x,y)=x^3+xy$ with respect to x, given that $y=\sin x$.

13. Check whether the expression $yz\,dx+xz\,dy+xy\,dz$ is exact.

14. Find the extremum value of the function $f(x,y)=x^3y^2(1-x-y)$ for $x\neq 0,\ y\neq 0$.

15. The temperature T at any point $(x,y,\ z)$ in space is $T=400\,xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$.

16. Find the moment of inertia of a uniform triangular lamina of mass M with sides *a* and *b about one of the sides of length a.*

17. If R is the region bounded by the circle $x^2+y^2=1$ in the first quadrant evaluate $\iint_R \frac{xy}{\sqrt{1-y^2}}\,dy\,dx$.

18. Find the Jacobian of U and V with respect to x and y if $u=x^3y$ and $v=e^x$.

19. Prove that the Beta integral is symmetric in its variables.

20. Prove that $\int_0^{\frac{\pi}{2}} \sin^p\theta\cos^q\theta\,d\theta=\frac{1}{2}\beta\left(\frac{p+1}{2},\frac{q+1}{2}\right)$.

21. Prove that the value of $\beta\left(3,\frac{1}{2}\right)$ is $\frac{16}{15}$.

22. Prove that if n is a positive integer $\Gamma\left(n+\frac{1}{2}\right)=\frac{1.3.5....(2n-1)}{2^n}\sqrt{n}$.

PART – C

Answer **any six** questions. Each questions carries **4** marks.

23. Verify $\frac{\partial^2 f}{\partial x\, dy} = \frac{\partial^2 f}{\partial y\, \partial x}$ for the functions $f(x, y) = 2x^3y^2 + y$.

24. If $xyz = 8$ find (*x, y, z*) at which the function $f = \frac{5xyz}{x + 2y + 4z}$ is a maximum.

25. Find the Taylor expansion upto the quadratic term in $(x - 2)$ and $(y - 3)$ for the function $f(x, y) = ye^x$.

26. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

27. Evaluate $\iiint_R xy\, dx\, dy\, dz$ where *R* is the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

28. If R is the region bounded by the planes $x = 0$, $z = 0, z = 0$ and the cylinder $x^2 + y^2 = 1$, evaluate the $\iiint_R xyz\, dx\, dy\, dz$ by changing it to cylindrical coordinates.

29. Prove that $\frac{\beta(m+1, n)}{m} = \frac{\beta(m, n+1)}{n} = \frac{\beta(m, n)}{m+n}$.

30. Show that $\Gamma(n)\Gamma(1-n) = \beta(n, (1-n)) = \int_0^1 \frac{x^n - 1}{1 + x} dx$ where $0 < x < 1$.

31. Prove that $\Gamma(n)\Gamma\left(n + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}}\Gamma(2n)$ for $n > 0$.

PART – D

Answer **any two** questions. Each questions carries **15** marks.

32. If $x = e^u \cos\theta$, $y = e^u \sin\theta$ show that $\frac{\partial^2 \phi}{\partial u^2} + \frac{\partial^2 \phi}{\partial \theta^2} = (x^2 + y^2)\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$ where $f(x, y) = \phi(u, \theta)$.

33. (a) Evaluate $\iint_R (x+y)^2 dx\,dy$ where R is the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(b) Evaluate $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dy\,dx$ by changing the order of integration.

34. (a) Using tripple integrals find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

(b) Find the volume of the sphere $x^2+y^2+z^2=a^2$ using integrals.

35. (a) Prove that $ya \neq b$, $\beta(m,n)=\frac{1}{(b-a)^{m+n-1}}\int_a^b (x-a)^{m-1}(b-x)^{n-1}dx$.

(b) Prove that $\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2n-1}}\Gamma(2n)$.

(Pages : 3) **M – 2405**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Physics

Complementary Course for Statistics

PY 1231.3 : THERMAL PHYSICS AND STATISTICAL MECHANICS

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A (very short answer type)

Answer **all** questions in **one** word or maximum of **two** sentences. **Each** question carries **1** mark.

1. Explain isothermal process.

2. State Dulong – petit law.

3. Write down the expression for work done by an ideal gas in an isothermal process.

4. Explain thermometric conductivity.

5. Write down the expression for adiabatic elasticity.

6. State Clausius statement of the second law of thermodynamics.

7. Write the expression for Planck's radiation formula.

P.T.O.

8. Is a proton a Fermion?

9. Define efficiency of an engine.

10. What is Pauli's exclusion principle?

(10 $\times$ 1 = 10 Marks)

SECTION – B (Short Answer)

Answer any **eight** questions in about one paragraph. **Each** question carries **2** marks.

11. Explain Fermi energy.

12. Describe the Rayleigh-Jeans formula.

13. Describe the TS diagram for Carnot cycle.

14. Explain the concept of ensemble.

15. Describe Stefan's law.

16. Discuss the change in entropy during an irreversible process.

17. What is the connection between entropy and disorder?

18. Compare average velocity, root mean square velocity and most probable velocity.

19. Derive the relation Cp – Cv = nR.

20. Explain the specific heat of electrons in metals.

21. Explain Carnot's theorem.

22. Distinguish between reversible and irreversible processes.

(8 $\times$ 2 = 16 Marks)

SECTION – C (Short Essay)

Answer any **six** questions. **Each** carries **4** marks.

23. It is claimed that a particular, engine absorbs 200 Joules of heat at 500 K, does 100 Joules of work and rejects 75 Joules of heat into a sink at 212 K. It is possible.

24. Find the change in entropy when three moles of a gas expands isothermally to thrice its initial volume.

25. A gas within a cyclinder with a pressure of 3 atoms at room temperature of 27°C suddenly bursts. Find its resulting temperature. Take γ = 1.4.

26. State and explain Wiedmann-Franz Law. Also define thermal conductivity.

27. Obtain Stefan-Boltzmann law from Planck's radiation formula.

28. Describe Wein's displacement law.

29. Explain the carnot cycle with suitable diagram.

30. Explain the change in entropy when ice is converted into steam.

31. Describe how the Rayleigh-Jeans law fails to explain the black body spectrum.

(6×4 = 24 Marks)

SECTION – D (Long Essay)

Answer any **two** questions. **Each** carries **15** marks.

32. Derive the expression for adiabatic process of an ideal gas. What is the work done by an ideal gas in an adiabatic process?

33. Explain Einstein's theory of specific heat of solids.

34. Describe the working of a Carnot engine and derive the expression for its efficiency.

35. Use Fermi-Dirac statistics to explain the specific heat of electrons in a metal.

(2×15 = 30 Marks)

(Pages : 4) **M – 2406**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Computer Science

Complementary Course for Mathematics and Statistics

CS 1231.2/CS 1231.3 : PROGRAMMING IN C

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – A [Very Short Answer Type]

One word to maximum of one sentence, Answer **all** questions. Each question carries **1** mark.

1. Define Pseudocode.

2. What do you mean by Programming paradigm?

3. Give the function of sizeof() operator.

4. What is typedef?

5. What does the file stdio.h consist of?

6. What are Library functions?

7. What is Conditional operator?

8. Define Scope of a Variable.

P.T.O.

9. What is Pointer?

10. Mention the role of Dynamic Memory Allocation.

(10 $\times$ 1 = 10 Marks)

SECTION – B [Short Answer]

Not to exceed **one** paragraph, Answer any **eight** questions. Each question carries **2** marks.

11. What are the various characteristics of Flowchart?

12. Define the term (a) identifier (b) token.

13. What is the syntax of nested if statement?

14. What are Local and Global variables?

15. What is significance of gets and puts functions.

16. What is array? How it is declared?

17. Give the advantages of function.

18. Write the usage of Pointers.

19. What is Format specifier?

20. Give any four String Functions.

21. Define Union.

22. Write a 'c' program to add two numbers using pointers.

23. Write any four Advantages of Pointers.

24. How Structure is represented?

25. Define enumeration.

26. Explain the meaning of following statement with reference to pointers.

```
int * a, b;
b = 20;
*a = b;
a = & b;
```

(8 × 2 = 16 Marks)

SECTION – C [Short Essay]

Not to exceed **120** words, Answer any **six** questions. Each question carries **4** marks.

27. What is the structure of a C program?

28. Distinguish between variable and constant.

29. Describe the use of for loop with syntax and flowchart.

30. Explain how 'switch' statement is used in the programs instead of 'if-else' statement with a suitable example program.

31. Write a C program to swap contents of two variables using call by reference.

32. Explain various Character input and output functions with the help of suitable examples.

33. Write the use of comma operator in for loop.

34. What are the various jumping statements? Explain.

35. Write a program to declare structure book having data member as book_name, book_id, book_price, author. Accept this data for three books and display it.

36. Explain the concept of Passing array to functions.

37. What is operator precedence and associativity? Explain.

38. Describe I/O statements in C language.

(6 × 4 = 24 Marks)

SECTION – D [Long Essay]

Answer any **two** questions. **Each** question carries **15** marks.

39. Discuss various Control Structures with relevant examples.

40. Explain in detail the Datatypes available in C languages with the help of examples.

41. Compare all three-loops available in C language.

42. Compare Recursion and iteration. Also write a Recursive program to find LCM and GCD of two numbers.

43. Explain Dynamic Memory Allocation in detail with examples.

44. Explain nesting of Structure with example. Define a structure data type called Timestruct containing 3 members called hour, minute and second. Develop a program that would assign values to the individual members and display the time in the form HH:MM:SS.

(2 × 15 = 30 Marks)

(Pages : 6) **M – 2407**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Foundation Course

ST 1221 – STATISTICAL METHODS II

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

Use of calculator is permitted

SECTION – A

(Very short answer)

Answer **all** questions. Each question carries **1** mark.

1. Square of correlation coefficient is known as ————.

2. If the variables X and Y are independent what is the value of regression coefficient?

3. What is the effect of change of origin and scale on regression coefficient?

4. The value of correlation ratio varies from ————.

5. If A and B are two attributes with (AB) =300 and expectation of (AB) = 250, state the nature of association between A and B.

P.T.O.

6. Name the function used to group a column in excel.

7. Define data warehouse.

8. Who introduced the term OLAP?

9. What is meant by logistic regression?

10. What is assign function in *R*?

(10 × 1 = 10 Marks)

SECTION – B

(Short answer)

Answer **any eight** questions. Each question carries **2** marks.

11. Define Karl Pearson correlation coefficient.

12. Define partial correlation with an example.

13. Comment on the following :

    For a bivariate distribution, the regression coefficient of Y on X is 4.2 and regression coefficient of X on Y is 0.5

14. Write down the normal equations (adopting principle of least squares) for second degree parabola fitting.

15. Point out the uses of probable error of correlation coefficient.

16. Why there are two regression lines in a bivariate distribution?

17. Write down the formula for multiple correlation coefficient in terms of total correlations.

18. If Karl Pearson correlation coefficient is 0.6 for 64 pairs of observations, find the probable error of correlation coefficient.

19. What is the limitation of data analysis in excel?

20. Write down the plot functions to create bar chart and pie chart in R.

21. How to fit a straight line in R?

22. What is classification in data mining?

23. How can you compare neural network and decision tree?

24. Mention the usage of time series in data mining.

25. What is discriminant analysis in data mining?

26. How to create histogram in R?

($8 \times 2 = 16$ Marks)

SECTION – C

(Short essay questions)

Answer **any six** questions. Each question carries **4** marks.

27. Show that correlation coefficient is independent of change of origin and scale.

28. What are regression coefficients? Show that coefficient of correlation is the geometric mean between regression coefficients.

29. Describe the scatter diagram method of studying correlation.

30. Comment on the following :

    (a) If the correlation coefficient is 0.8, it implies that 80% of the data are explained

    (b) Probable error of correlation coefficient may be used to determine the limits for population correlation coefficient.

31. In a bivariate distribution, the following values are obtained:

$$\sum X = 30,\ \sum Y = 40,\ \sum XY = 214,\ n = 5,\ \sum X^2 = 220,\ \sum Y^2 = 340.$$

Obtain the regression line of Y on X.

32. Explain the method of fitting of an exponential curve $y = ab^x$ by principle of least squares.

33. The lines of regression of a bivariate distribution are :

$8X - 10Y + 66 = 0$

$40X - 18Y - 214 = 0$

The standard deviation of X is 3. Find the mean values of X and Y. What is the coefficient of correlation between X and Y?

34. How to calculate quartile deviation, mean deviation and variance using *R* software?

35. Describe the role of link analysis in data mining technique.

36. Write down the *R* commands to fit lines of regression and to compute regression coefficients.

37. What is clustering? What are the different clustering techniques?

38. Explain briefly different stages of data mining.

(6 × 4 = 24 Marks)

SECTION – D

(Essay questions)

Answer **any two** questions. Each question carries **15** marks.

39. (a) Define rank correlation. State and establish Spearman's rank correlation coefficient.

(b) Calculate Spearman's rank correlation for the following data :

X	115	22	148	251	83	47	325	92	70	164
Y	84	385	200	110	292	152	86	120	301	144

(8 + 7 = 15)

40. (a) Explain different types of correlation with examples. How do you interpret a calculated value of correlation coefficient?

(b) Explain the concept of regression and point out its uses.

(c) Prove that regression line of Y on X intersect at $(\overline{X}, \overline{Y})$ **(6 + 4 + 5 = 15)**

41. (a) What is meant by association of attributes? Define Yule's coefficient of association.

(b) The residents of a village, who were interviewed during a sample survey are classified below according to their smoking and tea drinking habits. Calculate Yule's coefficient of association and comment on its value.

	Smokers	Non-smokers
Tea drinkers	40	33
Non-tea drinkers	3	12

(c) With the usual notations if $r_{12} = 0.59$, $r_{23} = 0.77$, and $r_{13} = 0.46$ find $R_{1.23}$ and $r_{12.3}$. **(4 + 5 + 6 = 15)**

42. (a) How is data mining different from knowledge discovery in data base?

(b) Distinguish between sequence mining and spatial mining.

(c) Describe the role of data mining in data ware house.

(d) Define predictive data mining. How can regression be used as a data mining tool? **(3 + 3 + 4 + 5 = 15)**

43. (a) Describe the essential features of decision trees. How is it useful to classify data?

(b) What is nearest neighbor rule? What are the characteristics of K nearest neighbor algorithm?

(c) What is logistic regression in data mining? Why do we use logistic regression for classification?

(6 + 5 + 4 = 15)

44. (a) Describe the commonly used methods of data input in *R*.

(b) Explain the computations of mean, median for a frequency distribution using *R* software.

(10 + 5 = 15)

(2 $\times$ 15 = 30 Marks)

(Pages : 6) **M – 2408**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Mathematics

Complementary Course for Statistics

MM 1231.4 : MATHEMATICS II — ADVANCED DIFFERENTIAL AND INTEGRAL CALCULUS

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. (Each question carries **1** mark)

1. Find f_{xy} if $f(x, y) = 2x^3y^2 + y^3$.

2. Find $\frac{\delta f}{\delta x}$ and $\frac{\delta f}{\delta y}$ if $f(x, y) = y(\exp(x + y))$.

3. Show that $xdy + 3ydx$ is inexact.

4. Write the Taylor expansion of a function $f(x, y)$ of two variables.

5. Evaluate $\int_0^1\int_0^1 x^2 ydydx$.

P.T.O.

6. Evaluate $\int_0^1\int_0^2\int_0^1 dx\,dy\,dz$.

7. Evaluate $\dfrac{\Gamma(10)}{\Gamma(8)}$.

8. Write the integral expression for $\Gamma(p)$.

9. Evaluate the beta function $B(4, 1)$.

10. Express $\int_0^\infty \dfrac{x^3 dx}{(1+x)^5}$ as a beta function.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. (Each question carries **2** marks)

11. Find the total derivative of $f(x, y) = x^2 + 3xy$ with respect to x, given that $y = \sin^{-1} x$.

12. Find the total differential of the function $f(x, y) = y \exp(x+y)$.

13. Write the chain rule for partial differentiation if $f(x, y)$ is a function in x, y and both x, y are functions of another variable u.

14. Find the rate of change of $f(x, y) = xe^{-y}$ with respect to u if $x(u) = 1+au$ and $y(u) = bu^3$.

15. Define stationary point of two variable function. How to determine whether it is maximum or minimum?

16. Find the volume of the tetrahedron bounded by the three co-ordinate surface $x = 0$, $y = 0$ and $z = 0$ and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

17. Find the centre of mass of the solid hemisphere bounded by the surfaces $x^2 + y^2 + z^2 = a^2$ and the xy-plane, assuming that it has a uniform density ρ.

18. State Pappu's second theorem.

19. A semicircular uniform lamina is freely suspended from one of its corners. Show that its straight edge makes an angle of 23° with the vertical.

20. Find the moment of inertia of a uniform rectangular lamina of mass m with sides a and b about one of the sides of length b.

21. A tetrahedron is bounded by the three coordinate surfaces and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and has density $\rho(x, y, z) = \rho_0 \left(1 + \frac{x}{a}\right)$. Find the average value of the density.

22. Define gamma function and show that $\Gamma(n+1) = n!$.

23. Evaluate $\Gamma\left(\frac{9}{4}\right)$.

24. Show that $x^2 dy - (y^2 + xy)dx$ is not exact.

25. Evaluate $B(6, 4)$.

26. Find $\int_0^\infty \frac{x^3 dx}{(1+x)^5}$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. (Each question carries **4** marks)

27. Derive the conditions for maxima, minima and saddle points for a function of two real variables.

28. Show that the function $f(x, y) = x^3 \exp(-x^2 - y^2)$ has a maximum at the point $\left(\sqrt{\frac{3}{2}}, 0\right)$ and minimum at $\left(-\sqrt{\frac{3}{2}}, 0\right)$.

29. The temperature of a point (x, y) on a unit circle is given by $T(x, y) = 1 + xy$. Find the temperature of the two hottest points on the circle.

30. Define beta function by a definite integral and show that $B(p, q) = B(q, p)$.

31. Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$.

32. Find the mass of the tetrahedron bounded by the three coordinate surface and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, if its density is give by $\rho(x, y, z) = \rho_0 \left(1 + \frac{z}{a}\right)$.

33. Explain the change of variable of variables in triple integrals.

34. Evaluate $\int\limits_{x^2+y^2=a^2}\int (a + \sqrt{x^2 + y^2}\, dxdy)$.

35. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

36. Evaluate $\dfrac{\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{5}{3}\right)}$.

37. Express $\int_0^{\infty} x^5 e - x^{x^2} dx$ as a gamma function.

38. Show that $\Gamma(p+1) = p\Gamma(p)$

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. (Each question carries **15** marks)

39. Transform the expression $\dfrac{\delta^2 f}{\delta x^2} + \dfrac{\delta^2 f}{\delta y^2}$ into one in ρ and ϕ. Where $x = \rho \cos\phi$ and $y = \rho \sin\phi$.

40. Find the stationary points of $f(x, y, z) = x^3 + y^3 + z^3$ subjected to the following constraints :

    (a) $g(x, y, z) = x^2 + y^2 + z^2 = 1$;

    (b) $g(x, y, z) = x^2 + y^2 + z^2 = 1$ and $h(x, y, z) = x + y + z = 0$.

41. Evaluate the integral $I = \int_{-\infty}^{\infty} e^{-x^2} dx$ in detail.

42. Find an expression for a volume element in spherical polar coordinates, and hence calculate the moment of inertia about a diameter of a uniform sphere of radius *a* and mass *M*.

43. (a) Prove that $B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

(b) Evaluate $\int_0^\infty \frac{y^2 dy}{(1+y)^6}$ using above expression.

44. Derive $B(p,\, q) = \int_0^\infty \frac{y^{p-1} dy}{(1+y)^{p+q}}$.

(2 × 15 = 30 Marks)

(Pages : 4) **M – 2409**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Physics

Complementary Course for Statistics

PY 1231.3 – THERMAL PHYSICS AND STATISTICAL MECHANICS

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions in a sentences or two, each carries **1** mark.

1. Define thermal conductivity.

2. Define Lorentz number.

3. Define adiabatic process.

4. What is adiabatic elasticity?

5. Define the Clausius statement of the second law of Thermodynamics.

6. What is the value of entropy in a reversible cycle?

7. What is latent heat?

8. What are bosons?

9. Write the Plank's quantization condition.

10. What is fermi energy?

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer **any eight** questions in paragraph, each carries **2** marks.

11. Explain thermometric conductivity.
12. Explain Weidman-Franz law.
13. What is ultraviolet catastrophe?
14. Explain any one application of Wien's displacement law.
15. Explain an isothermal process.
16. Explain the efficiency of a heat engine.
17. What are the parts of a heat engine?
18. Explain the second law of thermodynamics.
19. What do you mean by the heat death of Universe?
20. Define a TS diagram.
21. Explain the change in entropy when ice is converted to steam.
22. Entropy of the universe is always increasing. Justify this statement.
23. What is phase-space?
24. Define ensemble.
25. Explain how Planck 'law corrected the Rayleigh-jeans theory of radiation.
26. What are the theories of specific head of solids?

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions, each carries **4** marks.

27. Given that the thermal conductivity of the material of a slab is 10×10^{-2} $Wm^{-1} K^{-1}$ Calculate the amount of heat flows through the slab per second when the difference in temperature between the slab is 10 K. Given that the thickness of the slab is 1 cm and its area of cross section is 2×10^{-2} m^2.

28. The surface temperature of the Sun is 6000 K. Calculate the maximum wavelength which can be emitted from the Sun. Given that Wein's constant is 0.292×10^{-2} metre K.

29. Calculate the energy radiated by a blackbody at 1200K. Given that the Stefan's constant is 5.67×10^{-8} $W/m^2 K^4$.

30. Prove that the work done in an adiabatic change is equal to its change in internal energy.

31. Calculate the decrease in efficiency of a Carnot's engine works between 1000K and 300K changes it working temperature to 600 K and 300 K.

32. Calculate the entropy change when 2 kg of water at 373 K is converted to water vapour at: the same temperature. Given that the latent heat of vaporization of water is 2.26×10^6 JKg^{-1}.

33. Draw and explain the T-S diagram of a Carnot's cycle.

34. Calculate the energy of a radiation of wavelength 400 mm using Planck's distribution law.

35. Draw the black body spectrum and explain the energy density distribution as a function of temperature and frequency/wavelength,

36. Distinguish between canonical, micro canonical and grand canonical ensembles.

37. Calculate the velocity of a free electron at a temperature of 10 K. Given that the Boltzmann constant is 1.38×10^{-23} K^{-1}. Mass of the electron is 9.1×10^{-31} kg.

38. Derive the Einstein formula for the specific heat of solid.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions, each carries **15** marks.

39. With the help of a neat diagram and necessary theory explain the method of determination of Heat conductivity of a bad conductor by Lees' disc.

40. Derive the equation for work done in an isothermal process.

41. Explain the processes involving in a Carnot' s cycle. Derive the efficiency of a Carnot's engine.

42. (a) Derive the equation for the change in entropy in a irreversible process.

    (b) Derive the expression for entropy in a reversible isothermal process,

43. Compare Maxwell-Boltzmann, Fermi Dirac and B.E. Distribution laws.

44. Explain the theory of a blackbody. Derive the Rayleigh–Jeans law and the correction of Rayleigh law by –Planck's distribution law.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 2611**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022.

First Degree Programme under CBCSS

Mathematics

Complementary Course for Statistics

MM 1331.4 : MATHEMATICS – III

(Integration and Complex Numbers)

(2013 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Evaluate $\int_1^2 x\,dx$.

2. Distinguish between integrating velocity and integrating speed over a time interval.

3. Define the average value of a continuous function f on $[a, b]$.

4. Write the formula to find the arc length of the curve $y = f(x)$ over $[a, b]$.

5. Define a polar rectangle.

P.T.O.

6. Integrate $\int_0^1 \int_0^1 y\,dx\,dy$.

7. Estimate $\int \frac{\cos x}{\sin x} dx$.

8. When x is real, define $\cos h\,x$.

9. Find the principal argument of $-i$.

10. Write the conjugate of $i-2$.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Suppose that a curve $y = f(x)$ in the xy-plane has the property that at each point (x, y) on the curve, the tangent line has slope x^2. Find an equation for the curve given that it passes through the point (2, 1).

12. If a particle moves along a coordinate line so that its velocity at time t is $v(t) = 2 + \cos t$, find the average velocity of the particle during the time interval $0 \le t \le \pi$.

13. Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval [1, 4] is revolved about the x – axis.

14. Evaluate : $\int (x^2 + 1)^{50}\, 2x\,dx$.

15. Determine the value of $\int \sin(x-9)\,dx$.

16. Integrate : $\int_0^{\pi/3} \int_0^{\cos y} x \sin y\,dx\,dy$.

17. Compute : $\int_0^1 \int_{-x}^{x^2} y^2 x \, dy \, dx$.

18. Estimate : $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) \, dx \, dy \, dz$.

19. Find the period of the periodic function $\tanh z$.

20. Prove that $\frac{\pi}{8} = \frac{1}{1 \cdot 3} + \frac{1}{5 \cdot 7} + \frac{1}{9.11} + \cdots$

21. Separate into real and imaginary parts : $\sin h(x + iy)$.

22. Express $1 - i$ in polar form.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. **Each** question carries **4** marks.

23. Suppose that a particle moves with velocity $v(t) = \cos \pi t$ along a coordinate line. Assuming that the particle has coordinate s = 4 at time t = 0, find its position function.

24. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between x = 0 and x = 1 about the x – axis.

25. Integrate : $\int \sin^2 x \cos x \, dx$.

26. Evaluate $\iint_R y^2 x \, dA$ over the rectangle $R = \{(x, y) : -3 \le x \le 2, \ 0 \le y \le 1\}$.

27. Use a polar double integral to find the area enclosed by the three-petaled rose $r = \sin 3\theta$.

28. Compute : $\int_{0}^{2\pi}\int_{0}^{3}\int_{0}^{\sqrt{25-r^2}} r\,dz\,dr\,d\theta$.

29. Expand $\cos 5\theta$ in terms of powers of $\cos\theta$.

30. Find the cube roots of 1.

31. If $z_1 = i$ and $z_2 = -1+i$, then verify whether $\arg(z_1 z_2) = \arg(z_1)+\arg(z_2)$.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. **Each** question carries **15** marks.

32. A bus has stopped to pick up riders, and a woman is running at a constant velocity of 5 *m/s* to catch it. When she is 11 *m* behind the front door the bus pulls away with a constant acceleration of 1 m/s^2. From that point in time, how long will it take for the woman to reach the front door of the bus if she keeps running with a velocity of 5 *m/s*?

33. Find the area of the region enclosed by $x = y^2$ and $y = x - 2$.

34. (a) Verify whether $\int_{1}^{3}\int_{2}^{4}(40-2xy)\,dy\,dx = \int_{2}^{4}\int_{1}^{3}(40 = 2xy)\,dx\,dy$.

(b) Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes z = 1 and $y = x - 2$.

35. If $x = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ where θ is real, prove that $\theta = -i\log\tan\left(\frac{\pi}{4}+i\frac{x}{2}\right)$

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 2612**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Statistics

MM 1331.4 MATHEMATICS III – FOURIER SERIES, NUMERICAL METHODS AND ODE

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** the questions.

1. Define a periodic function.

2. Show that product of two odd function *s* even.

3. Define Fourier series expansion of a 2π-perodic function $f(x)$ in $(-\pi,\pi)$.

4. Find the integrating factor of $\frac{dy}{dx}+\cot x\, y = co\sec x$

5. Write the Auxiliary equation of $y''-2y'+y=0$.

6. Find the general solution of $y''+y=0$

7. Find the Wronskian of sin *x* and cos *x.*

P.T.O.

8. Write down the necessary condition for differential equation $M(x,y)dx + N(x,y) + dy = 0$ to be exact.

9. Give an example for transcendental equation

10. Find the augmented matrix of the system $x + y = 2, x + 2y = 1$.

(10 $\times$ 1 = 10 Marks)

SECTION II

Answer any **eight** questions.

11. Find the Fourier coefficient b_n of the function $f(x) = \begin{cases} 0 & if \quad -\pi \leq x \leq 0 \\ 1 & if \quad 0 \leq x \leq \pi \end{cases}$.

12. Define Fourier series of a 2 π periodic even function $f(x)$.

13. State Parseval Theorem.

14. Solve $(x+1)\frac{dy}{dx} + y = 3$.

15. Solve the differential equation $y'' + 3y' + 2y = 0$.

16. Construct a differential equation whose general solution is $y = Ae^{x} + Be^{-x}$.

17. Solve $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$.

18. Solve Clairaut's equation $y = px + p^2$.

19. Find the particular integral of $y'' + 3y' + 2y = e^{x}$.

20. Solve the system $x - y = 7, 2x - 8y = 8$ using Gaussian elimination.

21. Find an approximate root of $f(x) = x^2 - 3x + 2$ near zero, using Newton Raphson formula.

22. Write the Runge Kutta 4^{th} order formula.

(8 × 2 = 16 Marks)

SECTION III

Answer any **six** questions.

23. Represent function $f(x) = |x|; -\pi \le x \le \pi$ as a Fourier series.

24. Find the Fourier Transform of $f(x) = 1$ if $|x| < 1$ and $f(x) = 0$ otherwise.

25. Solve $xdy - ydx = \sqrt{x^2 + y^2}dx$.

26. Using Method of Variation of Parameter solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$.

27. Find the general solution of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x\cos x$.

28. Convert the given differential equation $(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0$ in to exact and solve for the particular solution if $y(0) = -1$.

29. Using Newton Raphson method find a recurrence formula to calculate square root of a positive integer N and hence find $\sqrt{12}$.

30. Find approximate value of $I = \int_0^1 \frac{dx}{1+x}$ by splitting the interval [0,1] in 8 equal parts.

31. Solve $x^2 \frac{d^2y}{dx^2} + 0.6x\frac{dy}{dx} + 16.04y = 0$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions.

32. (a) Use *Binary chopping* Method to find an approximate root of $f(x) = x^3 + 4x^2 - 10$ between [1, 2].

(b) Use Newton Raphson Method to find approximate root of $f(x) = x^3 + 4x^2 - 10$ with initial approximation $x = 1$

33. Solve $y'' + 2y' + 5y = e^{0.5x} + 40\cos 10x - 190\sin 10x$ given $y(0) = 0.16$ and $y'(0) = 40.08$

34. Solve

(a) Solve $\dfrac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$

(b) Solve $(e^y + 1)\cos x dx + e^y \sin x dy = 0$

(c) Solve $\dfrac{dy}{dx} + y\tan x = y^3 \sec x$.

35. Find half range Fourier sine and cosine series of $f(x) = \begin{cases} \dfrac{2k}{L}x & \text{if } 0 < x < \dfrac{L}{2} \\ \dfrac{2k}{L}(L - x) & \text{if } \dfrac{L}{2} < x < L \end{cases}$.

(2 × 15 = 30 Marks)

(Pages : 4) **N – 2613**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Physics

Complementary Course for Statistics

PY 1331.3 : PHYSICAL AND MODERN OPTICS AND ELECTRICITY

(2013 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions in **one** or **two** sentences. **Each** question carries **1** mark.

1. Do optical and geometrical path lengths be the same? Explain.

2. Define fringe width in interference.

3. What is meant by the resolving power of an optical instrument?

4. Why do wide slits not give diffraction with visible light?

5. What are the characteristics of a laser beam?

6. Define permeability.

7. What is the importance of form factor?

8. How does atomic magnetic dipole moment arise?

P.T.O.

9. What are susceptance and admittance?

10. What is the significance of Neel temperature?

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions, not exceeding a paragraph. **Each** question carries **2** marks.

11. Differentiate between interference caused by wavefront division and amplitude division.

12. List the features of the interference fringes.

13. What are Haidinger fringes? What are the requirements necessary for producing them?

14. What are the assumptions made by Fresnel to explain diffraction?

15. Differentiate between single and double slit Fraunhofer diffraction pattern.

16. Give the nature of the Fresnel's diffraction at a circular aperture.

17. Explain the necessity of cladding in optical fibers.

18. Explain any two applications of lasers.

19. Prove that the average value of ac voltage for half cycle is $0.637E_0$ where E_0 is the peak voltage.

20. Why do transformers be used for long distance power transmission?

21. What is magnetic susceptibility? Explain mass and molar susceptibility and give the relation connecting them.

22. Explain ferromagnetism.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. Mathematically prove the origin of the interference by the superposition of 2 waves.

24. Determine the thickness of a soap film (n = 1.33), which creates constructive 2^{nd} order interference in reflected light of 600 nm if the incidence is normal.

25. A narrow slit illuminated by light of 500 nm is placed at a distance of 8 cm, from a straight edge. The diffraction pattern is obtained at 1 m from the straight edge. Calculate the separation between the first and second dark bands.

26. Show that the intensities of first three maxima of diffraction patterns due to Fraunhofer diffraction at a single slit are in the ratio 1:1/22: 1/61.

27. The critical angle for a fibre made up of silica is 41°. Find the refractive index of the fiber. Also find the critical angle when the fibre is immersed in water (n_{water} = 1.33).

28. In a series LCR circuit with E = 220 V, f = 50 Hz, R = 200 Ω, L = 20 mH, C=$1\mu F$, find the maximum current and resonant frequency.

29. The magnetic field intensity in a piece of ferric oxide is 3×10^{6} A/m. If the susceptibility of the material is 10^{-3}, calculate the magnetisation of the material, relative permeability and the flux density.

30. Find the ratio of the populations of the two states in a laser that produces a light of wavelength 488 nm at 27°C.

31. A circuit consists of a resistance and capacitance in series. An alternating emf of 220 V, 50 Hz is applied to it. Find the value of resistance and capacitance when the maximum current is 5A and the active power is 500W.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. Discuss the measurement of wavelength and refractive index of the liquid using Newton's rings arrangement.

33. Discuss the theory of plane transmission grating. Explain the method of determination of wavelength of a spectral line using the grating.

34. Discuss the flow of ac through (a) LR series (b) CR series circuits. Draw the corresponding vector diagrams.

35. Discuss the propagation of light in optical fibers. Explain the distinguishing features and applications of step index and graded index optical fibers.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 2614**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Physics

Complementary Course for Statistics

PY 1331.3 : OPTICS, MAGNETISM AND ELECTRICITY

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions in **one** or **two** sentences. **Each** question carries **1** mark.

1. What are the types of interference?

2. Define coherent sources.

3. What is Fraunhoffer diffraction?

4. What is meant by grating element?

5. Give the full form of LASER.

6. What is the principle of fibre optics?

7. Write the relation between relative permeability and susceptibility.

8. Define intensity of magnetization.

P.T.O.

9. Define mean value of A.C.

10. What is the resonant frequency in LCR circuit?

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions, not exceeding a paragraph. **Each** question carries **2** marks.

11. Distinguish between interference and diffraction.

12. Give the difference between prism and grating spectra.

13. Define magnetic flux density (B) and magnetic field intensity (H) and give the relationship.

14. Explain magnetic susceptibility.

15. Explain power factor.

16. What is population inversion?

17. What is step index fibre?

18. What are the properties of diamagnetic materials?

19. What is interference in thin film?

20. Describe various losses occurring in a transformer.

21. Explain interference of light.

22. Why Newton's rings are circular?

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. In Newton's rings experiment the diameter of certain order of dark ring is measured to be double that of second ring. What is the order of the ring?

24. Distinguish between ferromagnetism and antiferromagnetism.

25. Explain peak, mean, r.m.s. and effective values of A.C.

26. The volume susceptibility of a magnetic material is 30×10^{-4}. Calculate the relative permeability. What is the nature of the substance?

27. A capacitor of capacitance $2\,\mu F$ is in an AC circuit of frequency 1000 Hz. If the rms value of the applied emf is 10 V, find the effective current flowing in the circuit.

28. Find the natural frequency of a circuit containing an inductor of $50\,\mu H$ and capacitor of capacitance $0.001 \mu F$.

29. Consider a step index fibre with $n1 = 1.461$, $n2 = 1.458$ and $a = 5mm$. Show that the fibre is single mode for $\lambda_0 > 1.22 \mu m$.

30. A transformer has a primary coil with 1600 loops and a secondary coil with 1000 loops. If the current in the primary coil is 6 Ampere, then what is the current in the secondary coil.

31. In a Newton's rings experiment the diameter of the 15^{th} ring was found to be 0.59 cm and that of the 5^{th} ring is 0.336 cm. If the radius of curvature of the lens is 100 cm, find the wave length of the light.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. Explain the formation of Newton's rings. How can it be used to determine the wavelength of a monochromatic light?

33. Describe the phenomenon of Fraunhoffer diffraction at a single slit.

34. Explain the principle, construction and working of Ruby laser.

35. Explain para and diamagnetic substances on the basis of electron theory.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 2616**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Statistics

Core Course II

ST 1341 : PROBABILITY AND DISTRIBUTION – I

(2013 – 2017Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Find the supremum and infimum of $\{-2,-3/2,-4/3,-5/4,\ldots\}$.

2. Let A be closed set and B be an open set. Then $B-A$ *is*… ———— set.

3. State Cauchy's general principle of convergence.

4. Define alternating series.

5. What do you mean by subsequence?

6. If A is a subset of B, then probability $P(A|B)$ is ————.

7. What is a probability space?

8. Define mutually exclusive events.

P.T.O.

9. When do you say that random variable is continuous?

10. Define marginal probability mass function.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Prove that 0 is the only limit point of the set $S = \left\{ \frac{1}{n} : n \in N \right\}$

12. State Cauchy's second theorem on limits.

13. Examine the convergence of the series $\sum_{1}^{\infty} \frac{1}{n(n+1)}$.

14. Explain Raabe's test.

15. State Leibnitz's test on alternating series.

16. What do you meant by divergent sequence?

17. Define random experiment.

18. What are the properties of joint distribution function?

19. Let X be a continuous random variable with pdf $(x) = \begin{cases} kx, 0 \le x \le 1 \\ k, \ 1 \le x \le 2 \\ 0, \textit{otherwise} \end{cases}$. Find the value of k.

20. Give axiomatic definition of probability.

21. A distribution function $F(x)$ is defined as $F(x) = \begin{cases} 0, & x \le 1 \\ \frac{1}{16}, (x-1)^4, 1 < x \le 3 \\ 1, & x > 3 \end{cases}$ find the pdf.

22. If the joint probability mass function of X_1 and X_2 is $p(x_1, x_2) = \frac{x_1 + x_2}{24}, x_1 = 0, 1, 2, 3, x_2 = 1, 2$. Find the marginal pmf of X_1.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. What do you mean by monotonic sequence?

24. Show that the necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.

25. Show that the sequence $a_n = \dfrac{2n-7}{3n+2}$ converges to 2/3.

26. Discuss the convergence of $1 + \dfrac{2!}{2^2} + \dfrac{3!}{3^3} + \ldots$.

27. Explain absolute and conditional convergence.

28. If two dice are thrown, what is the probability that the sum is (a) greater than 8 and (b) neither 7 not 11?

29. State and prove addition theorem of probability.

30. The diameter of an electric cable, say X is assumed to be a continuous random variable with pdf, $f(x) = 6(1-x)$, $0 \le x \le 1$.

    (a) Check whether the above is a pdf.

    (b) Determine a number b such that $P(X < b) = P(X > b)$.

31. Let X be a random variable with pdf $f(x) = \begin{cases} ke^{-2x}, 0 < x < \infty \\ 0, elsewhere \end{cases}$

    (a) Find k

    (b) Obtain the pdf of $Y = X^2$

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) State and prove Bolzano Weirstrass theorem

(b) Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{}$.

33. (a) Explain Cauchy's condensation test.

(b) Using Cauchy's condensation test, discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n \log n}$.

34. (a) State and prove Baye's theorem

(b) In a factory machines A, B and C produce 2000, 4000 and 5000 items in a month respectively. Out of their output 5%, 3% and 7% are defective. From the factory's products one is selected at random and inspected. What is the probability that it is good? If it is good, what is the probability that it is from Machine C?

35. If (X,Y) has the joint density function $f(x,y)=\begin{cases} \frac{1}{8}(6-x-y), 0<x<2, 2<y<4 \\ 0, otherwise \end{cases}$

(a) Determine the marginal densities.

(b) Test the independence of X and Y.

(c) Compute $P(X<1, Y<3), P(X<1|Y<3)$

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 2617**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022.

First Degree Programme under CBCSS

Statistics

Core Course – 3

ST 1341 – PROBABILITY AND DISTRIBUTION – I

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Define random experiment.

2. What do you mean by disjoint events?

3. Define probability mass function (pmf).

4. State addition theorem of probability.

5. Define mathematical expectation of a continuous random variable.

6. Define geometric mean in terms of expectation.

7. If $V(X) = 16$, then obtain $V\left(\frac{2X-16}{4}\right)$.

P.T.O.

8. If X and Y are two independent random variables, then obtain the moment generating function (mgf) of $X - Y$.

9. Define cumulant generating function.

10. Define conditional probability distribution

(10×1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks

11. If $B \subset A$, then Show that $P(B) \leq P(A)$.

12. If A and B are two independent events in the sample space S. Show that A^C and B^C are also independent.

13. Define distribution function. Give any four properties.

14. A continuous random variable with pdf.

    $f(x) = kx^4,\ -1 < x < 0; = 0$ elsewhere. Find k

15. If the possible values of a random variable X are 0,1,2,... Show that $E(X) = \sum_{n=0}^{\infty} p(X > n)$.

16. Find the mgf *of* X with pdf $f(x) = e^{-x}, x > 0$.

17. The joint pdf of X and Y, $f(x, y) = Kx(y - x), 0 < x < 4, 4 < y < 8$. Find the value *K.*

18. Define bivariate moment generating function, Give the condition to get the univariate *mgf.*

19. For any two random variables X and Y, show that $E(X) = E[E(X / Y)]$.

20. Obtain the recurrence relation between raw moments and central moments using mathematical expectation.

21. Show that the mgf of the sum of two independent random variables is the product of their mgf's.

22. Let $\varphi(t)$ be the characteristic function of a continuous random variable X. then show that $|\varphi_X(t)| \le 1$.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks

23. If A, *B* and *C* are pair wise independent and A is independent of *BUG.* Prove that *A, B* and *C* are mutually independent.

24. A box contains 3 red and 7 white balls, One ball is drawn at random and in its place a ball of the other colour is put in the box. Now one ball is drawn at random from the box. Find the probability that it is red.

25. Given a discrete random variable X with pmf as

x:	0	1	2	3	4
P(x):	0.2	k	2k	$\frac{k}{2}$	0.1

(a) Find k

(b) Find $P(0.5 \le x \le 2.5)$.

26. If X is a continuous random variable with the pdf $f(x) = e^{-x}, 0 < x < \infty$ Find the pdfs of

(a) $Y = 2X + 5$ and

(b) $Y = X^3$.

27. State and prove Cauchy-Schwartz inequality.

28. Let X be a random variable with pmf $P(X = x) = p(1-p)^{x-1}$, $x = 1,2,3,...$ Find its moment generating function and hence the mean.

29. Given the joint probability masses of (X,Y) as $p(x,y) = \frac{1}{4}$ for $(x,y) = (1,1),(1,2),)(2,1),(2,2)$. Examine whether X and Y are independent.

30. Let $f(x,y)=e^{-x-y}, x>0, y>0$ be the joint pdf of (X,Y). Find the bivariate mgf and prove that X and Y are independent.

31. Define characteristic function. Prove any two properties of characteristic function.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

32. For a random variable X with possible values $-3,-2,-1,0,1,2,3$ and given $P(X=-3)=P(X=-2)=P(X=-1); P(X=3=P(X=2)=P(X=1)$ and $P(X=0)=P(X>0)=P(X<0)$. Obtain the probability mass function of X and its distribution function.

33. Let X be a random variable with pdf

$$f(x)=\begin{cases} kx, \textit{ when } 0<x<1 \\ k, \textit{ when } 1<x<2 \quad \textit{and } 0, \textit{ elsewhere}. \\ -kx+3k, \textit{ when} 2<x<3 \end{cases}$$

(a) Find k ,

(b) Find F(x), and

(c) Find E(X).

34. Give $f(x,y)=c(xy^2+e^x), 0<x<1, 0<y<1$ is a joint pdf

Find (i) c (ii) marginal pdfs of X and Y. (iii) P(1/2 <X<2/3) and (iv) P(X<1/2, Y<1/4).

35. The pdf of two random variables X and Y is given as

$f(x,y)=2, 0\le x\le y\le 1; 0$ *otherwise.* Find (i) Coefficient of correlation between X and Y. (ii) $V(X/Y)=y)$.

(2 $\times$ 15 = 30 Marks)

(Pages : 6) **N – 2618**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Mathematics

Complimentary Course for Statistics

MM 1331.4 : MATHEMATICS III – FOURIER SERIES, NUMERICAL METHODS AND ODE

(2019 and 2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions.

1. Define a periodic function.

2. Show that product of two odd function *s* even.

3. Define Fourier series expansion of a 2π-perodic function $f(x)$ in $(-\pi, \pi)$

4. Find the integrating factor of $\frac{dy}{dx} + \cot x\, y = \text{cosecx}$.

P.T.O.

5. Write the Auxiliary equation of $y'' - 2y' + y = 0$.

6. Find the general solution of $y'' + y = 0$.

7. Find the Wronskian of $\sin x$ and $\cos x$.

8. Write down the necessary condition for differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ to be exact.

9. Give an example for transcendental equation.

10. Find the augmented matrix of the system $x + y = 2$, $x + 2y = 1$.

(10 $\times$ 1 = 10 Marks)

SECTION – II

Answer any **eight** questions.

11. Find the Fourier coefficient b_n of the function $f(x) = \begin{cases} 0 & \text{if} \quad -\pi \le x \le 0 \\ 1 & \text{if} \quad 0 \le x \le \pi \end{cases}$.

12. Define Fourier series of a 2π periodic even function $f(x)$.

13. State Parseval Theorem.

14. Solve $(x+1)\dfrac{dy}{dx} + y = 3$.

15. Solve the differential equation $y'' + 3y' + 2y = 0$.

16. Construct a differential equation representing a family of concentric circles centred at (0, 0).

17. Construct a differential equation whose general solution is $y = Ae^{x} + Be^{-x}$.

18. Solve $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$.

19. Solve Clairaut's equation $y = px + p^{2}$.

20. Find the particular integral of $y'' + 3y' + 2y = e^{x}$.

21. Define recurrence relation with an example.

22. Find the complimentary function of the differential equation $y'' - 8y' + 20y = x$.

23. Solve the system $x - y = 7$, $2x - 8y = 8$ using Gaussian elimination.

24. Find an approximate root of $f(x) = x^{2} - 3x + 2$ near zero, using Newton Raphson formula.

25. Write the steps in Gaussian elimination method to solve a system of linear equation.

26. Write the Runge Kutta 4th order formula.

(8 $\times$ 2 = 16 Marks)

SECTION – III

Answer any **six** questions.

27. Represent function $f(x) = |x|$; $-\pi \le x \le \pi$ as a Fourier series.

28. Find the Fourier Transform of $f(x) = 1$ if $|x| < 1$ and $f(x) = 0$ otherwise.

29. Solve $xdy - ydx = \sqrt{x^2 + y^2}\, dx$.

30. Find integrating factor of the differential equation $2xydy = (x^2 + y^2)dx$ and hence find its solution.

31. Using Method of Variation of Parameter solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$.

32. Solve $\frac{dx}{3x^2} - \frac{dy}{(3xy + y^2)} = 0$.

33. Find the general solution of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x\cos x$.

34. Convert the given differential equation $(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0$ in to exact and solve for the particular solution if $y(0) = -1$.

35. Using Newton Raphson method find a recurrence formula to calculate square root of a positive integer N and hence find $\sqrt{12}$.

36. Find approximate value of $I = \int_0^1 \frac{dx}{1+x}$ by splitting the interval [0, 1] in 8 equal parts.

37. Solve $x^2\frac{d^2y}{dx^2} + 0.6x\frac{dy}{dx} + 16.04y = 0$.

38. Evaluate using Trapezium rule $I = \int_1^2 \frac{dx}{5+3x}$ by splitting interval to 4 equal parts.

(6 $\times$ 4 = 24 Marks)

SECTION – IV

Answer any **two** questions.

39. (a) Use *Binary chopping* Method to find an approximate root of $f(x) = x^3 + 4x^2 - 10$ between $[1,2]$.

(b) Use Newton Raphson Method to find approximate root of $f(x) = x^3 + 4x^2 - 10$ with initial approximation $x = 1$.

40. (a) Using Gaussian elimination solve the system :

$$x + 10y - 3z = 3$$
$$2x + 3y + 20z = 7$$
$$10x + 3y + 2z = 4$$

(b) Find Solution of the system of equation

$$45x_1 + 2x_2 + 3x_3 = 58$$
$$-3x_1 + 22x_2 + 2x_3 = 47$$
$$5x_1 + x_2 + 20x_3 + 67$$

using Gauss-Seidel Method.

41. Solve $y'' + 2y' + 5y = e^{0.5x} + 40\cos 10x - 190\sin 10x$ given $y(0) = 0.16 \,\&\, y'(0) = 40.08$

42. Solve

(a) Solve $\dfrac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$

(b) Solve $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$

(c) Solve $\dfrac{dy}{dx} + y\tan x = y^3 \sec x$.

43. Find half range Fourier sine and cosine series of $f(x)=\begin{cases} \frac{2k}{L}x & if \quad 0<x<\frac{L}{2} \\ \frac{2k}{L}(L-x) & if \quad \frac{L}{2}<x<L \end{cases}$.

44. Find the Fourier series of 2-periodic function $f(x)=\begin{cases} 1 & for \quad 0\le x\le 1 \\ x & for \quad 1\le x\le 2 \end{cases}$

Hence deduce that

(a) $\sum_1^\infty \frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$

(b) $\sum_1^\infty \frac{(-1)^{n+1}}{2n-1}=\frac{\pi}{4}$.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 2619**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Physics

Complementary Course for Statistics

PY 1331.3 : OPTICS, MAGNETISM AND ELECTRICITY

(2019 & 2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** ten questions, **each** carries **1** mark.

1. Write any two examples for the occurrence of interference

2. What is a grating?

3. Whether diffraction can be produced by white light? Justify?

4. What do you mean by coherence?

5. What is population inversion?

6. Write any two properties of laser light.

7. Explain stimulated emission.

8. Define relative permeability of a magnetic material.

P.T.O.

9. What do you mean by paramagnetism?

10. Differentiate acceptor and rejecter LCR circuits.

($10 \times 1 = 10$ Marks)

SECTION – B

Answer any **eight** questions, **each** carries **2** marks.

11. How the dark and bright fringes are separated in Young's double slit experiment?

12. Explain the colours of thin films.

13. Why two independent monochromatic light sources could not produce interference?

14. Distinguish between Fresnel and Fraunhofer diffractions.

15. Distinguish between the spectra of a prism and of a grating.

16. What is optical pumping?

17. Why a two level system could not produce lasing?

18. What are the uses of optical fibres?

19. Explain Curie temperature.

20. Write the relations between susceptibility, permeability, M B and H of a magnetic material.

21. What is diamagnetism? What will be the value of susceptibility of a perfect diamagnetic material?

22. What is antiferrognetism?

23. Explain how the emfs in primary and secondary are related to the number of turns in the primary and secondary coils of a transformer?

24. Define the terms, impedance and admittance in an ac circuit.

25. Define Q factor of an ac circuit.

26. Explain wattles current.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions, **each** carries **4** marks.

27. In a Young's double slit experiment, when two waves of amplitudes a1 and a2 are interfering square of their resultant amplitude is $a^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$. Derive the expression for intensity distribution at interference condition, when $a_1 = a_2 = a$. Sketch the intensity distribution as a function of phase difference.

28. Calculate the fringe width in double slit experiment when the sources are separated by a distance of 1mm and the screen is placed at a distance of 1 m. Given that the wavelength used is 5890 nm.

29. Explain the formation of the first dark ring in the Fraunhofer diffraction by a circular aperture.

30. Calculate the number of lines per meter of a plane transmission grating if it can produce a second order line at a diffraction angle of 30 degrees if light of wavelength 542 nm is incidenting on it?

31. Calculate the temperature at which the spontaneous emission and the stimulated emission becomes equal. Given that the wavelength emitted is 600 nm.

32. *A* step index fibre has the following parameters. $n1 = 1.682$, $n2 = 1.442$ and $n_{air} = 1$. Calculate the critical angle and numerical aperture.

33. Explain any four applications of lasers.

34. A rod of magnetic material 0.5 m in length has a coil of 500 turns wounded over it uniformly. If a current of 1 A is passing through it, calculate the magnetic field H, the intensity of magnetization M, and magnetic induction B and the relative permittivity μ_r. Given that susceptibility $\chi_m = 6\times10^{-3}$.

35. Write the difference between diamagnetic, ferromagnetic and paramagnetic materials.

36. A circuit consists of a non-inductive resistance of 50 ohms, an inductor of 0.3 henry and a resistance of 2 ohms and a capacitor of 40 microfarads. Calculate the current, lag or lead and the power factor in the circuit. Given that supply is 50 Hz, 220 V.

37. Derive an expression for current in an ac circuit containing an inductance and a resistance in series. What is its impedance?

38. A parallel beam of light of wavelength 589 nm is incident on a thin glass plate of refractive index 1.5, such that the angle of refraction into the plate is 60 degrees. Calculate the smallest thickness of the glass plate which will appear dark by reflection.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions, **each** carries **15** marks.

39. Sketch the Newton's ring arrangement with necessary explanation. Derive an expression for the measurement of wavelength using Newton's ring apparatus.

40. Explain the Fresnel theory of rectilinear propagation of light.

41. Explain the theory of a transmission grating and obtain the condition for the n^{th} principal maxima. How a grating can be constructed and used to measure the wavelength of a light source?

42. Explain the propagation step index and graded index optical fibres. Explain the propagation of light through these fibres.

43. Explain ferromagnetism. Explain the domain theory and Weiss theory of ferromagnetism.

44. With necessary theory explain the production of emf in a coil rotating in a magnetic.

(2 $\times$ 15 = 30 Marks)

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(Pages : 6) **N – 2620**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Statistics

Core Course

ST 1341 : PROBABILITY & DISTRIBUTION – I

(2019 & 2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Define random variable.

2. If A and B are mutually exclusive events, what is $P(A \cap B)$?

3. What is the probability that a letter of English alphabet is chosen at random is a vowel?

4. Define mathematical expectation.

5. If $E(X)$ =12, what is $E(X+4)$?

6. Give an example of a discrete random variable.

7. If $P(s)$ is the probability generating function of X, what is the probability generating function of $X + 1$?

P.T.O.

8. What is the effect of change of origin and scale on moment generating function?

9. Suppose A and B events with $P(A) = 0.5$, $P(B) = 0.2$ and $P(AB) = 0.1$, what is the probability of either A or B occurs?

10. Prove that $P(A^c) = 1 - P(A)$.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Define classical definition of probability with an example.

12. A card is drawn at random from well shuffled pack of 52 cards. What is the probability that it is either a diamond or a king?

13. Define sample space. Write down the sample space for the random experiment of tossing a coin twice.

14. The diameter X of an electric cable is assumed to be a continuous random variable with probability density function $f(x) = 6x(1-x)$, $0 < x < 1$ and $f(x) = 0$, otherwise. Obtain the distribution function of X.

15. Define :

    (a) marginal distributions and

    (b) independence of random variables in continuous case.

16. Given $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, and $P\left(\frac{A}{B}\right) = \frac{1}{8}$. What is $P\left(\frac{B}{A}\right)$?

17. A random variable X takes values –1, 0 and 1 with respective probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. Find $Var(X)$.

18. Define characteristic function. Why does characteristic function always exist?

19. Let $f(x, y) = K$, for $0 < x < 1$, $x < y < x+1$ and $f(x,y) = 0$, otherwise. Find the value of K.

20. Distinguish between probability density function and probability mass function.

21. Let $f(x, y) = e^{-(x+y)}$, $0 < x < \infty$, $0 < y < \infty$ be the joint pdf of (X, Y). Examine whether X and Y are independent or not.

22. Three coins are tossed simultaneously. Find the probabilities of the following:
    (a) at least one head and
    (b) two heads.

23. Find the mean of a random variable X having moment generating function $M_X = (t) = (1-2t)^{-2}$.

24. State the properties of distribution function.

25. Distinguish between pairwise independence and mutual independence events.

26. Define geometric probability. Why is geometric probability important?

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. Show that conditional probability satisfies the axioms of probability.

28. State and prove addition theorem on probability.

29. Define the following terms with examples :
    (a) Random experiment
    (b) event,
    (c) equally likely events, and
    (d) mutually exclusive events.

30. State and prove multiplication theorem on expectation for discrete and continuous cases.

31. Suppose that the life in hours of a certain part of radio tube is a continuous random variable X with pdf given by $f(x) = \frac{100}{x^2}$, $x \geq 100$ and $f(x) = 0$, otherwise.

    Find the probability that a tube will last less than 200 hours given that the tube is still functioning after 150 hours.

32. Show that expected value of X is equal to the expectation of conditional expectation of X given Y.

33. Let X be a random variable with pdf $f(x)$. Give the definitions of median, mode, geometric mean and harmonic mean.

34. Define :
    (a) two dimensional distribution function
    (b) joint probability mass function and
    (c) conditional probability mass function.

35. Let X and Y are two random variables having joint pdf:

    $f(x, y) = \frac{1}{8}(6 - x - y)$; $0 < x < 2,\ 2 < y < 4$ and $f(x) = 0$, otherwise.

    Find $P(X < 1 / Y < 3)$.

36. A random variable X has the probability mass function $P(x) = \frac{1}{3}\left(\frac{2}{3}\right)^x$, $x = 0, 1, 2 \ldots$ Find the characteristic function of X.

37. A salesman has a 60% chance of making a sale to each customer. The behavior of successive customers is independent. If two customers A and B enter what is the probability that the salesman will make a sale to A or B?

38. Find the moment generating function of sample mean of n independently and identically distributed random variables.

($6 \times 4 = 24$ Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. (a) State and prove Bayes theorem on probability.

(b) A factory produces a certain type of outputs by three types of machines. The respective daily production are 3000, 2500 and 4500 units for machines A, B and C. Past experience shows that 1% of the output produced by machine A is defective. The corresponding fraction of defective for the other two machines B and C are 1.2% and 2% respectively An item is drawn at random from the day's production run and is found to be defective What is the probability that it comes from the output

(i) Machine A,

(ii) Machine B and

(iii) Machine C? **(6+9=15 Marks)**

40. (a) State and prove addition theorem on expectation.

(b) If X and Y are random variables, and for real constants a and b derive the expression for $Var\,(aX + bY)$.

(c) State and prove Cauchy-Schwartz inequality.

(3+5+7=15 Marks)

41. (a) Define cumulant generating function. State the properties of cumulants. Obtain the first four cumulants in terms of central moments.

(b) If $\phi_X(t)$ is the characteristic function of a random variable X, prove the following:

(i) $\phi_X(t)$ and $\phi_X(-t)$ are conjugate functions

(ii) $\phi_X(t)$ is a real valued and even function of t if the random variable X is symmetrical about zero. **(11+4=15 Marks)**

42. (a) The joint probability density function of (X,Y) is given by:

$f(x, y) = 2;\ 0 < x < 1,\ 0 < y < x$ and $f(x, y) = 0$, *otherwise*

(i) Find the marginal density functions of X and Y

(ii) Find the conditional density functions of Y given X and X given Y.

(iii) Check for independence of X and Y

(b) The pdf of random variable X is $f(x) = 30x^2(1-x)^2$, $0 < x < 1$ and $f(x) = 0$, otherwise. Find the distribution of $Y = X^2$. **(10+5=15 Marks)**

43. (a) Define :

(i) conditional expectation and

(ii) conditional variance.

The joint distribution of (X, Y) is $P(1, 1) = 0.1$, $P(2, 1) = 0.1$, $P(3, 1) = 0.2$, $P(1, 2) = 0.2$, $P(2, 2) = 0.3$ and $P(3, 2) = 0.1$. Find $E(Y / X = 3)$ and $V(X / Y = 1)$.

(b) Define covariance in terms of expectation. State its properties.

(11+4=15 Marks)

44. (a) State and prove multiplication theorem on probability.

(b) If A and B are independent, show that $\overline{A}$ and $\overline{B}$ are independent

(c) From a city population the probability of selecting a male or a smoker is 7/10, probability of a male smoker is 2/5 and probability of a male given that a smoker is already selected is 2/3.

Find the probability of selecting

(i) a non - smoker,

(ii) a male, and

(iii) a smoker, if a male is first selected. **(4+3+8=15 Marks)**

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 7827**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

STATISTICS

Core Course

ST 1441 : PROBABILITY AND DISTRIBUTION – II

(2013–2017 Admission)

Time : 3 Hours Max. Marks : 80

Use of Scientific Calculator and Statistical Table are permitted.

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. If X and Y are two random variables and a and b are constants, find $E(aX + bY)$.
2. State multiplication theorem on expectation.
3. State Cauchy – Schwartz inequality.
4. Define Probability generating function.
5. What is Bernoulli trial?
6. State a property of characteristic function.
7. Name a distribution with mean and variance equal.
8. For a poisson distribution $P(X = 1) = 2\ P(X = 0)$, what is the mean of X.
9. Name the discrete distribution having lack of memory property.
10. Define Gamma distribution.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer any **8** questions. Each carries **2** marks.

11. Express the variance of a random variable in terms of expectation.

12. Show that if x and y are independent random variables, $Cov\ (x,\ y) = 0$.

13. Find the mgf of a distribution with probability function $f(x) = 1/\theta,\ 0 < x < \theta,\ \theta > 0$.

14. Define correlation. Give the expression for the correlation in terms of Expectation.

15. Comment on the skewness and Kurtosis of Binomial distribution.

16. 1 and 2 are the modes of a Poisson variable. Find $P(X = 0)$.

17. Define Multinomial distribution.

18. Discuss the importance of Normal distribution in Statistics.

19. Write down the probability function of Normal variate and Standard Normal variate.

20. Find mean of Exponential random variable X with pdf $f(x) = \theta e^{-\theta x}, x > 0,\ \theta > 0$.

21. If X and Y are Independent Normal variables with mean μ and standard deviation σ, give the distribution of $Z = X + Y$.

22. If X is a standard normal variable, find $P(X > 0)$ and $P(X < 0)$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **6** questions. Each carries **4** marks.

23. Define and give the expression for Conditional Expectation and conditional variance.

24. State and prove addition theorem on expectation.

25. If a random variable X has mgf $M_x(t)$, find the mgf of $Y = aX + b$ and $Z = 3X - 4$.

26. If X is a Poisson variate with parameter λ find mode of X also find mean and variance of $Y = \frac{x-\lambda}{\sqrt{\lambda}}$.

27. If the random variable x has mgf $M_x(t) = \left(\frac{1}{3} + \frac{2e^t}{3}\right)^{10}$ find the first three central moments.

28. State and prove lack of memory property of Exponential distribution.

29. If X and Y are independent random variables with probability function $f(x) = 2e^{-2x} x > 0$ and $f(y) = e^{-y} y > 0$, find the distribution of $Z = X + Y$.

30. X is Normal with mean 20 and standard deviation 5. Find (i) $P(16 \le x \le 22)$ (ii) $P(X \ge 23)$.

31. If X is Normal with mean 6 and variance 49 and if $P(3x+8 < a) = P(4x-7 > b)$ and $P(5x-2 < b) = P(2x+1 > a)$ find a and b.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **2** questions. Each carries **15** marks.

32. (a) A random variable has distribution $f(x)$=1/2x for 0<x<1 and 0 otherwise. Find $E(X)$.

    (b) If $E(X)$=10 and $V(X)$=25, find positive values of a and b such that Y=aX–**b** has mean 0 and Variance 1.

    (c) If X follows Uniform distribution in (a, b), Find $E(X)$.

33. Define moment generating function. Let the random variable X has the distribution $P(X = r) = q^{r-1}p; r = 1,2,3,...$ Find the mgf and hence its mean and variance.

34. (a) State and prove additive property of Binomial distribution.

(b) Find the mode of the random variable X following Binomial distribution, $B(n,0.5)$, when n is odd and even.

(c) Derive the recurrence relation for Binomial probabilities.

35. (a) Define Uniform distribution and find its mean and variance.

(b) State 5 properties of Normal distribution.

(c) Comment on the points of inflection of Normal curve.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 7828**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

Statistics

Core Course

ST 1441 – PROBABILITY AND DISTRIBUTION II

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. If $X \sim B\left(100, \frac{1}{2}\right)$, find the mean of X.

2. Variance of a poisson variable is 4. Find $P(X = 1)$.

3. Define negative binomial distribution.

4. Write down the mean and variance of geometric distribution.

5. Define double exponential distribution.

6. State the additive property of gamma distribution.

7. Write down the harmonic mean of beta distribution of first kind with parameters p and q.

P.T.O.

8. What are two points of inflexion of normal distribution?

9. Write down any one property of Cauchy distribution.

10. If X (with p components) is distributed according to $N(\mu, \Sigma)$, then what is the distribution of $Y = CX$ where C is non singular.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **8** questions. Each question carries **2** marks.

11. If the m.g.f. of a random variable X is $\left(\frac{1}{3}+\frac{2}{3}e^{t}\right)^{5}$, find $P(X=2)$.

12. The mean and variance of a binomial random variable are 12 and 4 respectively. Find the parameters.

13. For a random variable X following Poisson distribution with parameter λ, the coefficient of variation is 50%. Find the value of λ.

14. The chance of a student to win a test is 0.2 and assumed remains same for all attempt. If he decides to compete until success, what is his chance to win the test by fifth attempt?

15. Write down the m.g.f. of geometric distribution and obtain the first row moment using it.

16. If $X - U(a, b)$ with mean 6 and variance 3. Find a and b.

17. Obtain the mean of beta distribution of 1^{st} kind.

18. Obtain the m.g.f. of exponential distribution.

19. Average weight of students in a class is 45 Kgs with a standard deviation of 9 Kgs. What is the probability of a student having weight greater than 48 kgs.

20. Find the distribution of $Y = aX + b$ when X is normal.

21. Obtain the r^{th} moment of log normal distribution.

22. Define multivariate normal distribution.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **6** questions. Each question carries **4** marks.

23. Five hundred families each having 4 children were taken as a sample. If the probability of a child being boy is 0.5, in how many families would you expect to have (a) exactly one boy (b) exactly two girls.

24. If X ~P(λ), prove that P(X is even) $= \frac{1}{2}\left[1 - e^{-\lambda}\right]^2$.

25. State and prove the lack of memory property of a Geometric distribution.

26. If X is a continuous random variable, find the distribution of $Y = F(x)$ where $F(x)$ is the distribution function of X.

27. Obtain the mean and variance of a Gamma distribution.

28. If X follow beta distribution of the first kind with parameters p and q. show that $X/(1-X)$ follow beta distribution of the second kind.

29. Obtain the m.g.f. of a Normal distribution.

30. Write down the properties of log normal distribution.

31. Show that the characteristic function of X distributed according to $N(\mu, \Sigma)$ is $e^{it'\mu - \frac{1}{2}t'\Sigma t}$.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **2** questions. **Each** question carries **15** marks.

32. (a) Obtain the m.g.f. hence mean and variance of negative binomial distribution.

    (b) Derive the conditional distribution of bivariate normal distribution and identify the distribution.

33. (a) Define a hyper geometric distribution and obtain the mean and variance of a hyper geometric distribution with parameters N, M and n.

(b) Show that hyper geometric distribution tends to binomial distribution as $N \to \infty$ and $\frac{M}{N} \to P$.

34. (a) X is a random variable with p.d.f. $f(x) = \lambda e^{-\lambda x}, x > 0$. Show that $y = 1 - e^{-\lambda x}$ is uniformly distributed over [0,1].

(b) The time taken by a mechanic to repair motorbike is distributed exponentially with an average of 2 hours. Find the probabilities of repair time (i) less than 1 hour (ii) more than 4 hours given that it already exceeds 2 hours.

35. (a) Fit a normal distribution to the following data and find the theoretical frequencies.

Class :	60-65	65-70	70-75	75-80	80-85	85-90	90-95	95-100
Frequency :	3	21	150	335	326	135	26	4

(b) For a normally distributed population 7% of the items have their value less than 35 and 87% have their values less than 63. Find the mean and standard deviation of the distribution.

(2×15 = 30 Marks)

(Pages : 6) **N – 7829**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Statistics

MM 1431.4 : MATHEMATICS IV – LINEAR ALGEBRA

(2013 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. If $u\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ and $v = \begin{bmatrix} 6 \\ -4 \\ 0 \end{bmatrix}$, find $4u - 5v$.

2. Find the norm of the vector $\begin{bmatrix} 1 \\ -2 \\ 2 \\ 0 \end{bmatrix}$.

3. Check whether the vectors $u = \begin{bmatrix} 9 \\ -4 \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix}$ are orthogonal.

P.T.O.

4. Find the inverse of $A = \begin{bmatrix} 3 & -9 \\ 2 & 6 \end{bmatrix}$.

5. Find the elementary row transformation that transforms A to B where $A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{bmatrix}$.

6. Find the rank of the matrix $\begin{bmatrix} 1 & -3 & 6 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.

7. Find the matrix of the quadratic form $5x^2 - 4xy + 7y^2$.

8. Find the image of $u = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ under the transformation $T : \mathcal{R}^2 \to \mathcal{R}^3$ be defined by $T(x) = Ax$, where $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$.

9. Write the standard matrix corresponding to the two dimensional shear transformation.

10. What is the order of the matrix of the transformation $T : \mathcal{R}^5 \to \mathcal{R}^6$?

(10 $\times$ 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. They carry **2** marks each.

11. Let $V = \{(x, y) : x \geq 0, y \geq 0\}$. Is V a vector subspace of $\mathcal{R}^2$ over $\mathcal{R}$? Justify your answer.

12. Define basis of a subspace of $\mathcal{R}^n$.

13. Let H be the set of vectors of the form $\begin{bmatrix} s+2r \\ 4s \\ -7r \end{bmatrix}$. Find a basis for H is $\mathcal{R}^3$.

14. Check whether the vectors $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ -9 \end{bmatrix}$ form a basis of $\mathcal{R}^3$. Justify your answer.

15. Find AB if $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$.

16. If $A = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$, show that $A^2 - 6A + 10I = 0$.

17. If 2 is an eigen value of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, find all the eigen values of A^5 without using characteristics equation.

18. By reducing to echelon form, find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \end{bmatrix}$.

19. Find the $\mathcal{B}$ – co-ordinate vector of x where $\mathcal{B} = \{b_1, b_2\}$, $b_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $x = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

20. Check whether $T : \mathcal{R}^3 \rightarrow \mathcal{R}^3$ given by $T[(x_1, x_2, x_3)] = (x_1, 0, x_3)$ is a linear transformation.

21. Check whether $T : \mathcal{R}^4 \to \mathcal{R}^4$ given by $T[(x_1, x_2, x_3, x_4)] = (0, x_1 + x_2, x_2 + x_3, x_3 + x_4)$ is (a) one-one (b) onto.

22. Find a basis for the null space and the column space of the matrix $\begin{bmatrix} -2 & -6 \\ -1 & 3 \\ -4 & 12 \\ 3 & -9 \end{bmatrix}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. They carry **4** marks each.

23. Find values of h, if any, such that $\{v_1, v_2, v_3\}$ is linearly dependent where, $v_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}$, $v_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}$.

24. Verify Cauchy Schwarz inequality for the vectors $u = (2, 3, 1), v = (-1, 4, 0)$.

25. Show that $\mathcal{B} = \{b_1, b_2, b_3\}$ form a basis of $\mathcal{R}^3$ where $b_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, $b_2 = \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}$, $b_3 = \begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix}$.

26. Solve the system by Cramer's rule

$$x + y - z = 9$$
$$8y + 6z = -6$$
$$-2x + 4y - 6z = 40$$

27. For what value of λ and μ do the system of equations

$$2x + 4y + 6z = 20$$
$$2x + 2y + 2z = 12$$
$$x + 2y + \lambda z = \mu$$

has (a) no solution (b) unique solution (c) infinite solution.

28. Make a change of variable $X = Py$ that transforms the quadratic form $3x_1^2 - 4x_1x_2 + 6x_2^2$ into one with no cross product term.

29. Let $\mathcal{B} = \{b_1, b_2\}$, $\mathcal{C} = \{c_1, c_2\}$ be bases of $\mathcal{R}^2$, where $b_1 = \begin{bmatrix} -1 \\ 8 \end{bmatrix}$, $b_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$, $c_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $c_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find the change of co-ordinate matrix from B to C and the change of co-ordinate matrix from C to B.

30. Write the matrix of the following transformations and draw the image of unit square under this transformation (a) reflection through x_1–axis (b) Reflection through the line $x_2 = -x_1$.

31. Find the $\mathcal{B}$ –matrix of the transformation $T : \mathcal{R}^2 \to \mathcal{R}^2$ given by $T(x) = Ax$, where $A = \begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix}$, $b_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathcal{B} = \{b_1, b_2\}$.

(6 $\times$ 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. They carries **15** marks each.

32. Using Gram Schmidt orthogonalization process obtain an orthonormal basis for $\mathcal{R}^3$ from the basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \right\}$

33. Prove that the matrix $\begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$ is diagonalisable and find P such that $A = PDP^{-1}$, where D is diagonal.

34. Prove that the vectors $u = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, w = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ are linearly dependent and find the linear dependence relation between them.

35. Identify the conic represented by the equation $3x_1^2 + 2x_1x_2 + 3x_2^2 - 8 = 0$ and find the equation of its major and minor axes.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 7830**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course of Statistics

MM 1431.4 – MATHEMATICS IV – LINEAR ALGEBRA

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions.

1. Define null space of a matrix A.

2. Write a basis other than standard basis for $\mathbb{R}^2$.

3. Define linear dependence of vectors in $\mathbb{R}^n$.

4. What can you say about the eigen values of an $n \times n$ symmetric matrix A if the quadratic form $X^T AX$ is negative definite?

5. Could a 6×9 matrix have a two dimensional null space?

6. True of False : If S spans V and if T is a subset of V that contains more vectors that S, then T is linearly dependent.

7. True of False : The only 3-dimensional subspace of $\mathbb{R}^3$ is $\mathbb{R}^3$ itself.

8. What is the dimension of the vector space $\mathbb{R}^n$ over $\mathbb{R}$?

P.T.O.

9. What is the size of the matrix of a linear transformation from $\mathbb{R}^5$ to $\mathbb{R}^6$ over $\mathbb{R}$?

10. If the null space of a 5×6 matrix A is 4 dimensional, what is the dimension of the column space of A.

(10×1 = 10 Marks)

SECTION – B

Answer **any eight** questions.

11. Prove that an $n \times n$ matrix with n distinct eigen values is diagonalizable.

12. Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Are $\vec{u}$ and $\vec{v}$ eigen vectors of A?

13. What are the eigen values of $A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$?

14. If $\vec{x}$ is an eigen vector for A corresponding to λ, what is $A^3\vec{x}$?

15. Find $T\left(a_0 + a_1 t + a_2 t^2\right)$, if T is the linear tranformation from $\mathbb{P}_2$ to $\mathbb{P}_2$ whose matrix relative to $\mathcal{B} = \{1, t, t^2\}$ is $\begin{bmatrix} 3 & 4 & 0 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix}$.

16. Find the standard matrix A for the dilation transformation $T(\vec{x}) = 3\vec{x}$ for $\vec{x}$ in $\mathbb{R}^2$.

17. Let T be the linear transformation whose standard matrix is $A = \begin{bmatrix} 1 & -4 & 8 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$.

Does T map $\mathbb{R}^3$ onto $\mathbb{R}^2$? Is T a one-to-one mapping?

18. Let $S = \{1, t, t^2, ..., t^n\}$. Verify that S is a basis for $\mathbb{P}_n$, the set of all polynomials of degree $\leq n$.

19. Determine the dimension of the supspace H of $\mathbb{R}^3$ spanned by the vectors $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ where $\vec{v}_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 3 \\ -7 \\ 1 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -1 \\ 6 \\ -7 \end{bmatrix}$.

20. What is the coordinate matrix of $(1, 7, 3)$ with respect to the standard basis $\{(1,0,0),(0,1,0),(0,0,1)\}$?

21. Define norm of a vector $\vec{v}$.

22. Show that $\vec{d}$ is orthogonal to $\vec{c}$ where $\vec{d} = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$ and $\vec{c} = \begin{bmatrix} 4/3 \\ -1 \\ 2/3 \end{bmatrix}$.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions.

23. Use Cramer's rule to solve the system
$3x_1 - 2x_2 = 6; -5x_1 + 4x_2 = 8$.

24. Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & 4 & -2 \end{bmatrix}$.

25. Find the characteristic polynomial and eigen values of the matrix $\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$.

26. Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k, given that $A = PDP^{-1}$.

27. Let $v_1 = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}, v_2 = \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}, v_4 = \begin{bmatrix} -4 \\ -8 \\ 9 \end{bmatrix}$ Find a basis for the subspace W spanned by $\{v_1, v_2, v_3, v_4\}$.

28. Check whether $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

29. Find a basis for the null space of the matrix $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$.

30. Determine whether the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

31. Is 5 an eigen value of $A = \begin{bmatrix} 6 & -3 & 1 \\ 3 & 0 & 5 \\ 2 & 2 & 6 \end{bmatrix}$?

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions.

32. Find an orthogonal basis for the column space of the matrix $\begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$.

33. Dragonalize the following matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$.

34. (a) If a vector space V has a basis $\mathcal{B} = \{b_1, ..., b_n\}$, then prove that any set in V containing more than n vectors must be linearly dependent.

(b) Find the dimension of the subspace $H = \left\{ \begin{bmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \text{ in } \mathbb{R} \right\}$.

35. (a) Let $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. Is 2 an eigen value of A? If so, find a basis for the corresponding eigen space.

(b) Prove that the eigen values of a triangular matrix are the entries on its main diagonal.

(2 × 15 = 30 Marks)

(Pages : 4) **N – 7831**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

Physics

Complementary Course for Statistics

PY 1431.3 : MODERN PHYSICS AND ELECTRONICS

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** the questions. **Each** carries **1** mark.

1. What is 2's complement? Give an example.

2. What do you mean by wave function?

3. What do you mean by the binding energy of the nucleus?

4. What do you mean by L-S coupling scheme?

5. What led to quantum mechanics?

6. What is the decimal equivalent of 10010?

7. What do you mean by rectification?

8. What is the nuclear decay constant?

P.T.O.

9. How is a n-p-n and p-n-p transistor is biased for normal operation?

10. What do you mean by depletion region?

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** carries **2** marks.

11. Explain the nuclear packing fraction.

12. Differentiate between Zener diode and Avalanche breakdown.

13. What is j-j coupling scheme?

14. Explain the periodic classification of elements.

15. What do you understand by Compton Effect?

16. Draw the logical symbol of an AND gate, an AND gate with two diode and give its truth table.

17. Compare the CE and CB transistor configurations.

18. Explain the need for biasing a transistor. What are the different methods of biasing?

19. Explain the fundamental concepts of Plank's theory.

20. Define Pauli's exclusion principle?

21. What is Bohr magneton? What is its significance?

22. What are the inadequacies of classical theory?

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. Calculate the time required for 10% of a sample of Thorium to disintegrate. Assume the half life of thorium to be 1.4 x 10^{10} years.

24. The input signal given to a a CE amplifier having a voltage gain of 150 is $Vi = 2\cos(15t + 3\pi)$. What is the corresponding output signal?

25. In a common emitter amplifier, the load resistance of the output circuit is 800 times the resistance of the input circuit. If $\alpha = 0.99$, calculate the voltage gain?

26. Calculate the de-Broglie wavelength of an electron moving with one fifth of the speed of light. Neglect relativistic effects. ($h = 6.63x10-34J.s; c = 3x10^8$ m/s, mass of electron $= 9\times10^{-31}$kg)

27. 27. A full-wave rectifier uses two diodes, the internal resistance of each diode may be assumed constant at 20 Ω. The transformer r.m.s. secondary voltage from centre tap to each end of secondary is 50 V and load resistance is 980 Ω. Find: (i) the mean load current (ii) the r.m.s. value of load current.

28. A half-wave rectifier is used to supply 50V d.c. to a resistive load of 800 Ω. The diode has a resistance of 25 Ω. Calculate a.c. voltage required.

29. Convert the binary numbers (1010111) and (1110011) into (a) decimal (b) hexadecimal equivalents

30. The wavelength of H α line is 6563 AU. Find the value of Rydberg constant.

31. In a transistor circuit, collector load is 4 kΩ Whereas quiescent current (zero signal collector current) is 1 mA. (i) What is the operating point if V_{cc}=10 V? (ii) What will be the operating point if R_c = 5 kΩ?

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** carries **15** marks.

32. Give an account of the Bohr model of the atom. Explain the origin of the spectral lines of hydrogen on the basis of this theory.

33. Briefly explain about binding energy of a nucleus, the features of the binding energy and the stability of the nucleus.

34. State and explain the De-Morgan's theorems. Prove them by the method of perfect induction illustrating the logical operations in a table.

35. With a neat diagram explain the working of a half wave rectifier, its efficiency and ripple factor.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 7833**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

Statistics

Core Course

ST 1441 : PROBABILITY AND DISTRIBUTION II

(2019 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define Bernoulli distributions.

2. Find the variance of binomial distribution with $n = 10$ and $P = 1/2$.

3. A Poisson distribution has a double mode at $X = 2$ and at $X = 3$. Find its mean.

4. Give the probability function of a multinominal distribution.

5. Give a discrete distribution which posess lack of memory property.

6. Write down the distribution function of exponential distributions with parameter θ.

7. Define standard normal variate.

P.T.O.

8. Define beta distribution of second kind.

9. Describe lognormal distribution.

10. What is the relation between mean, median and mode of normal distribution.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Find the moment generating function of discrete uniform distribution with n points.

12. Obtain the mean of hypergeometric distribution.

13. Let $X \sim P(\lambda)$ such that $P(X = 3) = P(X = 4)$. Find $P(X = 0)$.

14. Obtain the variance of a geometric distribution.

15. Find the probability generating function of poisson distribution.

16. Discuss the additive property of Bernoulli distribution.

17. Obtain the mean and variance of a degenerate distribution.

18. Discuss the applications of life testing problems.

19. Find the recurrence relation of probabilities of Poisson distribution.

20. Find the moment generating function of uniform $(0, \theta)$ distribution.

21. Define bivariate normal distribution.

22. Describe double exponential distribution.

23. Write any two properties of normal distribution.

24. Obtain the mean of log normal distribution.

25. Define Cauchy distribution. Explain its standard form.

26. Obtain the $P(X \leq 3)$ in the case of exponential distribution with $\theta = 1$.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

27. With usual notations show that the Poisson distribution is a limitting case of binomial distribution.

28. Obtain the first four raw moments of geometric distribution.

29. Establish the recurrence relation of central moments of binomial distribution.

30. State and prove lack of memory property of exponential distribution.

31. Obtain the moment generating function of $N(\mu, \sigma^2)$ and hence find its mean and variance.

32. Explain the properties of Cauchy distribution.

33. Find the mean and variance of beta distribution of first kind.

34. Obtain the variance-covariance matrix of bivariate normal distribution.

35. Explain the area property of normal distribution.

36. Establish the additive property of gamma distribution.

37. Obtain the mean and variance of double exponential distribution.

38. Find the mode of Poisson distribution.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. (a) Obtain the first four central moments of Poisson distribution and find the skewness and kurtosis.

    (b) Obtain the mean and variance of negative binomial distribution.

40. (a) Obtain the beta and gamma coefficients of normal distribution.

    (b) Establish the additive property of binomial distribution.

41. (a) Obtain the mean, variance and covariance of multinomial distribution.

    (b) Describe hypergeometric distribution and find its variance.

42. (a) obtain the mode of binomial distribution.

    (b) Find the mean deviation about mean of normal distribution.

43. Derive the marginal and conditional distribution of bivariate normal distribution.

44. (a) Obtain the moment generating function of gamma distribution and hence find its mean and variance.

    (b) Fit a Binomial distribution for the following data and obtain the expected frequencies. Also find the mean and variance of the fitted distribution.

x	0	1	2	3	4	5	6	7
$f(x)$	7	6	19	35	30	23	7	1

(2×15 = 30 Marks)

(Pages : 6) **N – 7834**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course of Statistics

MM 1431.4 – MATHEMATICS IV – LINEAR ALGEBRA

(2019 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Define null space of a matrix A.

2. Write a basis other than standard basis for $\mathbb{R}^2$.

3. Define linear dependence of vectors in $\mathbb{R}^n$

4. What can you say about the eigen values of an $n \times n$ symmetric matrix A if the quadratic form $X^T AX$ is negative definite?

5. Could a 6×9 matrix have a two dimensional null space?

6. True of False : If S spans V and if T is a subset of V that contains more vectors than S, then T is linearly dependent.

P.T.O.

7. True of False : The only 3-dimensional subspace of $\mathbb{R}^3$ is $\mathbb{R}^3$ itself.

8. What is the dimension of the vector space $\mathbb{R}^n$ over $\mathbb{R}$?

9. What is the size of the matrix of a linear transformation from $\mathbb{R}^5$ to $\mathbb{R}^6$ over $\mathbb{R}$?

10. If the null space of a 5×6 matrix A is 4 – dimensional, what is the dimension of the column space of A?

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each carries **2** marks.

11. Prove that an $n \times n$ matrix with n distinct eigen values is diagonalizable.

12. Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Are $\vec{u}$ and $\vec{v}$ eigen vectors of A?

13. What are the eigen values of $A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$?

14. If $\vec{x}$ is an eigen vector for A corresponding to λ, what is $A^3\vec{x}$?

15. Find $T\left(a_0 + a_1 t + a_2 t^2\right)$, if T is the linear transformation from $\mathbb{P}_2$ to $\mathbb{P}_2$ whose matrix relative to $\mathcal{B} = \{1, t, t^2\}$ is $\begin{bmatrix} 3 & 4 & 0 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix}$.

16. Find the standard matrix A for the dilation transformation $T(\vec{x}) = 3\vec{x}$ for $\vec{x}$ in $\mathbb{R}^2$.

17. Let T be the linear transformation whose standard matrix is $A = \begin{bmatrix} 1 & -4 & 8 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$.

Does T map $\mathbb{R}^4$ onto $\mathbb{R}^3$? Is T a one-to-one mapping?

18. Let $S = \{1, t, t^2, ..., t^n\}$. Verify that S is a basis for $\mathbb{P}_n$, the set of all polynomials of degree $\leq n$.

19. Determine the dimension of the subspace H of $\mathbb{R}^3$ spanned by the vectors $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ where $\vec{v}_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 3 \\ -7 \\ 1 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -1 \\ 6 \\ -7 \end{bmatrix}$.

20. What is the coordinate matrix of $(1,7,3)$ with respect to the standard basis $\{(1,0,0),(0,1,0),(0,0,1)\}$?

21. Define norm of a vector $\vec{v}$.

22. Show that $\vec{d}$ is orthogonal to $\vec{c}$ where $\vec{d} = \begin{bmatrix} 5 \\ 6 \\ -1 \end{bmatrix}$ and $\vec{c} = \begin{bmatrix} 4/3 \\ -1 \\ 2/3 \end{bmatrix}$.

23. Give an example of a subset of $\mathbb{R}^3$ that is not a subspace of $\mathbb{R}^3$.

24. If a set $S = \{v_1, v_2, ..., v_p\}$ in $\mathbb{R}^n$ contains the zero vector, then prove that the set S is linearly dependent.

25. Compute the quadratic form $X^T AX$ when $A = \begin{bmatrix} 5 & 1/3 \\ 1/3 & 1 \end{bmatrix}$ and $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

26. Show that $B = \{(3,0,0),(0,3,0),(0,0,3)\}$ is a basis for $\mathbb{R}^3$.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each carries **4** marks.

27. Use Cramer's rule to solve the system

$3x_1 - 2x_2 = 6; -5x_1 + 4x_2 = 8$.

28. Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & 4 & -2 \end{bmatrix}$.

29. Find the characteristic polynomial and eigen values of the matrix $\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$.

30. Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k, given that $A = PDP^{-1}$.

31. Let $v_1 = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}, v_2 = \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}, v_4 = \begin{bmatrix} -4 \\ -8 \\ 9 \end{bmatrix}$. Find a basis for the subspace W spanned by $\{v_1, v_2, v_3, v_4\}$.

32. Check whether $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

33. Find a basis for the null space of the matrix $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

34. Determine whether the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

35. Is 5 an eigen value of $A = \begin{bmatrix} 6 & -3 & 1 \\ 3 & 0 & 5 \\ 2 & 2 & 6 \end{bmatrix}$?

36. Let T: $\mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then prove that T is one-to-one if and only if $T(x) = 0$ has only the trivial solution.

37. Find the dimensions of the null space and the column space of $A = \begin{bmatrix} 1 & -6 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & 5 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

38. Let $T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)$. Show that T is a one-to-one linear transformation Does T maps $\mathbb{R}^2$ onto $\mathbb{R}^3$.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each carries **15** marks.

39. Find an orthogonal basis for the column space of the matrix $\begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$.

40. Diagonalize the following matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$.

41. (a) If a vector space V has a basis $\mathcal{B} = \{b_1, \ldots, b_n\}$, then prove that any set in V containing more than n vectors must be linearly dependent.

(b) Find the dimension of the subspace $H = \left\{ \begin{bmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \text{ in } \mathbb{R} \right\}$.

42. (a) Let $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. Is 2 an eigen value of A? If so, find a basis for the corresponding eigen space.

(b) Prove that the eigen values of a triangular matrix are the entries on its main diagonal.

43. Show that the mapping T: $\mathbb{P}_2 \to \mathbb{P}_2$ defined by $T(a_0 + a_1 t + a_2 t^2) = a_1 + 2a_2 t$ is a linear transformation. Also find the $\mathcal{B}$-matrix for T where $\mathcal{B}$ is the basis $\{1, t, t^2\}$.

44. Show that $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis for a subspace W of $\mathbb{R}^4$ and construct an orthonormal basis for W.

(2 × 15 = 30 Marks)

(Pages : 4) **N – 7835**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

Physics

Complementary Course for Statistics

PY 1431.3 : MODERN PHYSICS AND ELECTRONICS

(2019 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

(Answer **all** the questions. **Each** carries **1** mark)

1. What is 2's complement? Give an example?

2. What do you mean by wave function?

3. What do you mean by the Binding energy of the nucleus?

4. What do you mean by L-S coupling scheme?

5. What led to quantum mechanics?

6. What is the decimal equivalent of 10010?

7. What do you mean by rectification?

8. What is the nuclear decay constant?

P.T.O.

9. How is a n-p-n and p-n-p transistor is biased for normal operation?

10. What do you mean by depletion region?

(10 $\times$ 1 = 10 Marks)

SECTION – B

(Answer any **eight** questions. **Each** carries **2** marks)

11. Plot and explain the blackbody radiation spectrum at two temperatures T_1 and T_2 where $T_2 > T_1$.

12. Explain the nuclear packing fraction.

13. Define Pauli's exclusion principle.

14. What do you mean by ripple factor and efficiency of a rectifier?

15. Differentiate between Zener diode and Avalanche breakdown.

16. What is j-j coupling scheme?

17. Explain the periodic classification of elements.

18. What is Bohr magneton? What is its significance?

19. What are the inadequacies of classical theory?

20. What do you understand by Compton effect?

21. Draw the logical symbol of an AND gate and give its truth table .

22. Compare the CE and CB transistor configurations.

23. Why the NAND gate and NOR gate are called the universal gates?

24. Explain the need for biasing a transistor. What are the different methods of biasing?

25. Explain the fundamental concepts of Plank's theory.

26. What is meant by 'activity' of the radioactive material? Mention the different units of radioactivity.

(8 $\times$ 2 = 16 Marks)

SECTION – C

(Answer any **six** questions. **Each** carries **4** marks)

27. If 100.0 g of a radioactive isotope that has a half life of 25 years, identify the amount of that isotope that will remain after 100 years?

28. Calculate the time required for 10% of a sample of Thorium to disintegrate. Assume the half life of thorium to be 1.4×10^{10} years.

29. The input signal given to a CE amplifier having a voltage gain of 150 is $V_i = 2\cos(15t + 3\pi)$. What is the corresponding output signal?

30. In a common emitter amplifier, the load resistance of the output circuit is 800 times the resistance of the input circuit. If $\alpha = 0.99$, calculate the voltage gain?

31. Calculate the de-Broglie wavelength of an electron moving with one fifth of the speed of light. Neglect relativistic effects. ($h = 6.63\times10^{-34}$ J.s; $c = 3\times10^{8}$ m/s, mass of electron = 9×10^{-31} kg)

32. A full-wave rectifier uses two diodes, the internal resistance of each diode may be assumed constant at $20\,\Omega$. The transformer r.m.s. secondary voltage from centre tap to each end of secondary is 50 V and load resistance is $980\,\Omega$. Find: (a) the mean load current (b) the r.m.s. value of load current.

33. A half-wave rectifier is used to supply 50V d.c. to a resistive load of $800\,\Omega$. The diode has a resistance of 25 Ω. Calculate a.c. voltage required.

34. The applied input a.c. power to a half-wave rectifier is 100 watts. The d.c. output power obtained is 40 watts. (a) What is the rectification efficiency? (b) What happens to remaining 60 watts?

35. Convert the binary numbers (1010111) and (1110011) into (a) decimal (b) hexadecimal equivalents.

36. The wavelength of $H\alpha$ line is 6563 AU. Find the value of Rydberg constant.

37. In a transistor circuit, collector load is 4 $k\Omega$ whereas quiescent current (zero signal collector current) is 1 mA. (a) What is the operating point if Vcc = 10 V? (b) What will be the operating point if R_c = 5 $k\Omega$?

38. Convert the decimal number 110 into (a) octal and (b) hexadecimal

(6 $\times$ 4 = 24 Marks)

SECTION – D

(Answer any **two** questions. **Each** carries **15** marks)

39. Explain about the Load line and Q point of a transistor amplifier? Why do we prefer the Q-point in the middle of the active region? Explain graphically the operation of a transistor as an amplifier.

40. Deduce the time independent Schrodinger equation for a free particle.

41. Give an account of the Bohr model of the atom. Explain the origin of the spectral lines of hydrogen on the basis of this theory.

42. Briefly explain about binding energy of a nucleus, the features of the binding energy and the stability of the nucleus.

43. State and explain the De-Morgan's theorems. Prove them by the method of perfect induction illustrating the logical operations in a table.

44. With a neat diagram explain the working of a half wave rectifier, its efficiency and ripple factor.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **M – 1556**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course – V

ST 1541 : PROBABILITY AND DISTRIBUTION – III

(2014, 2016 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. What is the domain of the function $f(x) = \dfrac{1}{(x-1)(x-2)}$?

2. Write the p.d.f of a bivariate normal distribution.

3. Give an example of a function which is continuous every where but not differentiable at $x = 2$.

4. Give the Maclaurin series expansion of $\cos x$.

5. State the unit in which x should be expressed so that the Macluarin series expansion of $\sin x$ is valid.

6. If X and Y are independent normal variables with mean zero and variance 2 each, find the variance of $x - y$.

P.T.O.

7. What is meant by the continuity of a function $f(x)$ at $x = a$.

8. If $f(x) = \begin{cases} 2+x, x>0 \\ 2-x x<0 \end{cases}$ and $f(0) = k$, find the value of k if $f(x)$ is continuous at $x = 0$.

9. What is removable discontinuity?

10. Give the p.d.f. of distribution which has its characteristics function as $\phi(t) = \cos t$.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Define differentiability of a function at a point.

12. State Chebychev's inequality of and state the condition under which it is valid?

13. Give a continuous p.d.f. which has no m.g.f. but has characteristic function write that characteristic function.

14. Obtain the m.g.f. of a Gamma random variable with parameter $\alpha\, and \beta$.

15. If x follows $N(0,1)$, find the distribution of $Y = x^2$.

16. Define chi-square, F and t, distributions. State the relationship between F and t-distributions.

17. Find the value of θ in the mean value theorem for $f(x) = 3x^2 + 4x + 5$ in $[1,10]$.

18. State the weak low of large numbers.

19. State i.i.d. central limit theorem. Use it to show that binomial tends to normal.

20. If x follows Beta distribution of the first kind, find the probability distribution of $\frac{x}{1-x}$.

21. If (X,Y) is bivariate normal with parameters $(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$, find the conditional distribution X given Y.

22. Give the p.d.f. of a lognormal distribution.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

23. Prove that every differentiable functions are continuous.

24. Explain Rieman theory of integration of a function in $[a, b]$.

25. If X and Y are independent Poisson variables with parameters λ *and* μ, find the distribution of X given $X + Y = n$.

26. If X and Y are Binomial, state the conditions under which $x + y$ is Binomial and obtain the parameters of $X + Y$ under those conditions.

27. Define sampling distribution of a statistic. Obtain the sampling distribution of sample mean when sampling is from a normal $N(\mu, \sigma^2)$.

28. Two unbiased dice are tossed simultaneously. Let X be the sum of the numbers shown. Find the p.d.f. of X.

29. For a Beta distribution of the second kind with parameters p and q, find the upper limit of ‘r’ such that $E(x^r)$ exists.

30. State Lindberg-Levy form of the central limit theorem.

31. State and prove Maclarins theorem.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

32. (a) If *f(x)* is a pdf is [*0,a*] with distribution functions *F(x)* prove that $E(x) = \int_0^a [1 - F(x)]\, dx$.

    (b) Give a graphical method to find *E(x)* in the case pdf is [*0,a*].

33. If a random sample of size *'n'* is taken from $N(\mu, \sigma^2)$,show that sample mean $\overline{X}$ and sample variance s^1 are independently distributes. Find the p.d.f of s^2.

34. State and prove Taylors theorem. Obtain the Taylors series expansion of *Sin x* in terms of $x - \pi / 2$.

35. (a) State and prove additive property of chi-square distribution.

    (b) What is the relation between standard normal variable and a chi-square variable with n degrees of freedom?

    (c) Obtain the variance of chi-square distribution with *n* degrees of freedom.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **M – 1557**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Statistics

Core Course VI

ST 1542 — ESTIMATION

(2014, 2016 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define :

   (a) Parameter

   (b) Statistic.

2. Define most efficient estimator.

3. Give sufficient conditions for consistency.

4. Give 95% confidence limits for the variance of $N(\mu, \sigma)$, when μ is known.

5. State invariance property of sufficient estimator.

6. What is minimum variance bound estimator.

7. State any two properties of maximum likelihood estimator.

P.T.O.

8. Define moment estimator.

9. Define quadratic loss function.

10. Define risk function.

$(10 \times 1 = 10$ Marks)

SECTION – B

Answer **any eight** questions. **Each** carries **2** marks.

11. Let $X_1, X_2, X_3, \ldots. X_n$ is a random sample drawn from $N(\mu, 1)$. Obtain an unbiased estimator for $\mu^2 + 1$.

12. If T is consistent estimator for θ, then show that T^2 is consistent for θ^2.

13. Find the moment estimator for the parameter θ in Poisson distribution with density function, $p(x, \theta) = \frac{e^{-\theta}\theta^x}{x!}$, $x = 0, 1, 2, \ldots, \theta > 0$.

14. Describe estimation of parameters using maximum likelihood estimation.

15. Derive 95% confidence interval for the difference of population proportions for large samples.

16. Check whether Minimum Variance Bound (MVB) estimator exists for the parameter θ in the Cauchy population with pdf $f(x, \theta) = \frac{1}{\pi} \frac{1}{1 + (x - \theta)}$, $-\infty < x, \theta < \infty$.

17. How is Cramer-Rao inequality useful in obtaining MVUE?

18. Let $X_1, X_2, X_3, \ldots, X_n$ is a random sample drawn from a distribution with pdf $f(x, \theta) = e^{-(x-\theta)}$, $x \geq \theta$, $-\infty < \theta < \infty$. Obtain sufficient statistic for θ.

19. Let X be random variable with probability density function, $f(x, \theta) = (1 + \theta)x^{\theta}$, $0 < x < 1$. Find MLE of θ based on a random sample of n observations.

20. Describe the estimation of parameters using method of least squares.

21. Describe Baye's estimation.

22. What is 0 – 1 loss function? Give a situation where it is employed.

$(8 \times 2 = 16$ Marks)

SECTION – C

Answer **any six** questions. **Each** carries **4** marks.

23. What do you meant by point estimation? Give an example of an estimator

    (a) which is consistent but not unbiased,

    (b) which is unbiased but not consistent.

24. Let $X_1, X_2, X_3, ..., X_n$ be a random sample drawn from $N(\mu, \sigma^2)$ then obtain the maximum likelihood estimators for μ and σ^2.

25. Let X_1, X_2, X_3, X_4 be is a random sample of size 4 drawn from $N(\mu, \sigma^2)$. Let $t_1 = \frac{X_1 + X_2 + X_3 + X_4}{4}$, $t_2 = \frac{X_1 + 3X_2 + 2X_3 + X_4}{7}$, find efficiency of t_2 relative to t_1. Which is relatively more efficient estimator?

26. Describe the method of moments for estimating the parameters. What are the properties of the estimates obtained by this method?

27. Derive $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$ if sample sizes n_1 and n_2 are large.

28. Let $X_1, X_2, X_3, ..., X_n$ be a random sample drawn from $N(\mu, 1)$. Obtain the MVUE of μ.

29. Define sufficient estimator. Let x_1, x_2 be i.i.d. $P(\lambda)$ random variables then show that $x_1 + 2x_2$ is not sufficient for λ.

30. $X_1, X_2, ..., X_n$ is a random sample from a population with pdf $f(x) = \frac{1}{\theta}$, $0 < x < \theta$. Show that $X_{(n)} = \max(X_1, X_2, ..., X_n)$ is sufficient for θ and $\frac{(n+1)}{n} X_{(n)}$ is an unbiased estimator for θ.

31. Discuss the meaning and calculation of the posterior distribution in Bayesian analysis. Give an example to illustrate the procedure.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. **Each** carries **15** marks.

32. (a) Prove that in sampling from $N(\mu, \sigma)$ population, the sample mean is a consistent estimator for μ.

(b) Prove that for Cauchy sample mean is not consistent estimator but sample median is consistent for population mean.

33. Derive confidence interval for population mean when

(a) σ known

(b) σ unknown.

34. (a) State Cramer-Rao inequality and describe its regularity conditions,

(b) A random sample $X_1, X_2, ..., X_n$ is taken from a uniform population with pdf, $f(x) = 1,\ \theta - \frac{1}{2} \le x_i \le \theta + \frac{1}{2},\ -\infty < \theta < \infty$. Obtain the MLE for θ.

35. The sample values from population with pdf $f(x) = \theta x^{\theta - 1},\ 0 < x < 1,\ \theta > 0$, are given below :

0.46, 0.38, 0.61, 0.82, 0.59, 0.33, 0.72, 0.44, 0.59, 0.60.

(a) Derive moment estimate of θ

(b) Compute the moment estimate of the parameter θ for the given data.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **M – 1558**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course VII

ST 1543 : TESTING OF HYPOTHESIS

(2014, 2016-2017 Admission)

Time : 3 Hours Max. Marks : 80

(Use of statistical table and calculator are permitted)

SECTION – A

Answer **all** questions. **Each** carries **1** mark.

1. The size of a test is equal to the area of the ————

2. Level of significance lies between ———— and ————

3. Whether a test one sided or two sided depends on ————hypothesis.

4. To test an hypothesis about proportions of success in a class, the usual test is ————

5. Paired t test is applicable only when the observations are ————

6. For testing the equality of two population means, having the same variances, using t test the degrees of freedom in the useful notation is ————

P.T.O.

7. What is Test statistic?

8. When we use Non parametric tests?

9. Write the statistic of Wilcoxon signed rank test of one sample.

10. In a coin tossing experiment, the sample space is given by {H, H, T, H, H, T, T, H, H, T}. Find the number of runs.

(10×1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Distinguish between Simple and Composite hypotheses. Give one example each.

12. Explain Null hypothesis and Alternative hypothesis.

13. Distinguish between level of significance and power of the test.

14. State difference between Most powerful and Uniformly most powerful test?

15. State Neymann Pearson lemma.

16. Distinguish between large sample and small sample tests.

17. What is Yate's correction?

18. Explain the difference between unpaired t test and paired sample t test.

19. Explain chi square test for goodness of fit.

20. Explain parametric test and non-parametric test. Also give examples.

21. Define empirical distribution function.

22. Define Kernel U statistic.

(8×2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. Explain

    (a) Type I Error

    (b) Type II Error

    (c) Degrees of freedom

    (d) Critical region

24. It is desired to test a hypothesis H_0: p = 1/2 against the alternative hypothesis H_0: p = 3/4 on the basis of tossing a coin once, where p is the probability of getting a head in a single trial and agreeing to accept H_0 if a tail appears and to accept H_1 otherwise. Find the values of α and β.

25. Derive the most powerful test to test the mean of Binomial distribution by using Neymann Pearson lemma.

26. Given in the usual notation.

    $n_1 = 400$ $\bar{x}_1 = 250$ $s_1 = 40$
    $n_2 = 400$ $\bar{x}_2 = 220$ $s_2 = 55$

    Test whether the two samples have come from populations having the same mean (Use $\alpha = 0.01$).

27. Explain the *t* test for single mean.

28. Explain the Chi square test for variance.

29. Explain Wilcoxon signed rank test for one sample.

30. Explain the run test for randomness.

31. Explain the median test.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. Find the size and power of the following test, reject H_0: $\theta = 1$ in favor of H_1: $\theta = 2$ whenever $X_1 + X_2 \geq 2$. Assume that X_1 and X_2 are independent observation from Poisson (θ) distribution.

33. Derive the most powerful tests to test the mean and variance of normal population by using Neymann Pearson lemma.

34. Explain the Z test for

    (a) testing mean of a population

    (b) testing equality of two population mean

35. Explain the Kolmogorov-Smirnov test.

($2 \times 15 = 30$ Marks)

(Pages : 4) **M – 1559**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course VIII

ST 1544 : SAMPLE SURVEY METHODS

(2014, 2016-2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define simple Random sampling.

2. What is meant by sampling fraction?

3. Give an estimate of population total in SRSWOR.

4. What do you mean by census survey?

5. The total number of sample of size n = 2 from a population of N = 6 by SRSWR is.

6. Define finite population correction.

7. Define sampling error.

8. What is the ratio estimate of the population mean $\overline{Y}$?

P.T.O.

9. Describe Lottery method.

10. Define linear regression model.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Define Non-probability sampling.

12. Describe Random number method of selecting a SRS.

13. Show that under SRSWR P is an unbiased estimate of P?

14. Define (a) Sampling frame (b) Sampling units (c) Sampling design and (d) Statistics.

15. Explain the disadvantages of census survey.

16. Derive the confidence interval of population mean $\overline{Y}$ in SRSWOR.

17. Define systematic sampling.

18. Define stratified Random sampling.

19. Explain optimum allocation.

20. Show that $\overline{y}_{sy}$ is an unbiased estimate of population mean?

21. Show that ratio estimates are consistent estimators?

22. Explain the principles of sample survey.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. Distinguish between census and sample survey.

24. Distinguish between sampling and non-sampling error.

25. Describe the method of determining the sample size in sample proportion (SRSWOR).

26. Show that in SRSWOR the variance of sample mean is given by $\frac{N-n}{Nn}S^2$.

27. Write down the merits and demerits of stratified random sampling.

28. Compare the relative efficiency of stratified random sampling over SRS.

29. Explain circular systematic sampling.

30. Obtain the upper bound of the bias $(\hat{R})$?

31. Show that under SRSWOR, probability of getting a specified sample of 'n' units is equal to $\frac{1}{NC_n}$ or $\frac{1}{\binom{N}{n}}$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. Define sampling. Explain the principle steps in a sample survey.

33. S.T. $V(\bar{y})_{ran} \geq V(\bar{y})_{Pr\,op} \geq V(\bar{y})_{Neyman}$.

34. In SRSWOR, show that the sample variance $S^2 = \frac{1}{n-1},\sum_{i=1}^{n}(yi - \bar{y})^2$ is an unbiased estimate of population variance.

35. S.T. $V(\bar{y}_{lr}) = \frac{1-f}{n}\left(S_y^2 - 2b_o S_{yx} + b_o^2\, S_x^2\right)\bar{y}_{lr}$ denotes the linear regression estimate of $\bar{y}$.

(2 $\times$ 15 = 30 Marks)

(Pages : 7) **M – 1564**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course V

ST 1541 – LIMIT THEOREMS AND SAMPLING DISTRIBUTIONS

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

Use of scientific calculators and statistical tables are allowed.

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Using axioms of probability show that $P(A^c) = 1 - P(A)$.

2. Suppose $\{X_n\}$ and $\{Y_n\}$ converges to X and Y respectively in probability. What can you say about the convergence of $\{X_n \cdot Y_n\}$?

3. Define convergence in distribution of a sequence of random variables $\{X_n\}$.

4. What is the sample range of a random sample $X_1, X_2, ..., X_n$ drawn from a population?

P.T.O.

5. If $\overline{X}$ is the sample mean of a random sample drawn from a population with mean μ and variance σ^2, what is the Mean Square Error of $\overline{X}$?

6. If $X^2 \sim X_n^2$, where $n > 2$, obtain the point at which the probability density function of χ^2 attains maximum?

7. If $\chi_1^2 \sim \chi_{(3)}^2$ and $\chi_2^2 \sim \chi_{(5)}^2$ are two independent Chi-square random variables, what is the mean of $\chi_1^2 + \chi_2^2$?

8. If $t \sim t_{(4)}$ is a student's t variable with 4 degrees of freedom, using statistical table, find k such that $P(|t| \leq k) = 0.90$.

9. Let (X_1, X_2, X_3) be a random sample from $N(\mu, \sigma^2)$ then define a F statistic with (1, 2) degrees freedom using (X_1, X_2, X_3).

10. Define non-central F distribution.

(10 $\times$ 1 = 10 Marks)

SECTION – B

(Answer **any eight** Questions. Each question carries **2** marks)

11. Let $C_1, C_2, \ldots\ldots C_n, \ldots\ldots$ be a partition of a sample space and if A is any event, then prove that $P(A) = \sum_{i=1}^{\infty} P(A \cap C_i)$

12. Suppose $\{X_n\}$ is a sequence of random variables with probability mass function $P(X_n = 1) = \frac{1}{n}$ and $P(X_n = 0) = 1 - \frac{1}{n}$. Examine the convergence in probability of $\{X_n\}$.

13. Let $\{X_n\}$ be a sequence of random variables with distribution function $F_{X_n}(x) = 1 - \left(1 - \frac{x}{n}\right)^n$; $x > 0$ and 0 otherwise. Show that $\{X_n\}$ converges in distribution to an exponential distribution with unit mean.

14. If $\{A_n\}$, $n = 1, 2, \ldots.$ is a sequence of events defined over a probability space, define independence of events.

15. A random sample of size 64 are drawn from a population with mean 32 and standard deviation 5. Find the mean and standard deviation of the sample mean $\overline{X}$.

16. Let $X_1, X_2, \ldots. X_n, X_{n+1}$ be a random sample of size $n+1$ then show that $\overline{X}_{n+1} = \frac{X_{n+1} + n\overline{X}_n}{n+1}$, where $\overline{X}_{n+1}$ and $\overline{X}_n$ are the sample means of first $n+1$ and n observations respectively.

17. A random variable X has mean 5 and variance 3. Find the least value of $P(|X - 5| < 7.5)$ using Chebyshev's inequality.

18. Let $X_1, X_2, \ldots., X_{100}$ be a random sample of size 100 drawn from a population with mean 10 and variance 9, then find $P(\overline{X} > 10.5)$ using central limit theorem.

19. Let $(X_1, X_2, \ldots. X_n)$ be a random sample drawn from a population with distribution function $F(x.)$ Find the distribution function of smallest order statistic $X_{(1)}$.

20. Let $X_1, X_2, \ldots., X_{20}$ is a random sample of size 20 drawn from a normal population with mean μ and variance $\sigma^2 = 5$, if $s^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$ is the sample variance, find the mean of s^2.

21. If (X_1, X_2, X_3) is a random sample of size 3 from a standard normal population N (0,1), what is the sampling distribution of $U = \frac{\sqrt{2}\, X_3}{\sqrt{X_1^2 + X_2^2}}$.

22. Suppose $X \sim \chi^2_{(n)}$ and $Z = X + Y \sim \chi^2_{(m)}$ where X and Y are independent random variables and $m > n$. Write down the moment generating function of *X* and *Z*. Hence identify the distribution of *Y.*

23. Let $X_i \sim N(i, i^2)$, $i = 1, 2, 3$ are three independent normal random variables, then give an expression for *t* statistic with 2 degrees of freedom using X_1, X_2, X_3.

24. Using statistical table, find the left tailed critical values corresponding to area 0.05 for

    (a) Chi-square distribution with 10 degrees of freedom

    (b) *t* distribution with 15 degrees of freedom

25. Let $(X_1, X_2, ...X_n)$ be a sequence of independent normal random variables such that $X_i \sim N(\mu, \sigma_i^2)$, $i = 1, 2, ..., n$. Define a non-central Chi-square random variable $(X_1, X_2, ...X_n)$.

26. If $X \sim F(m, n)$ write down the probability density function of $\frac{1}{X}$.

(8 × 2 = 16 Marks)

SECTION – C

(Answer **any six** Questions. Each question carries **4** marks)

27. Explain sample space, sigma field and probability measure.

28. If $A_1, A_2, ..., A_{n...}$ is a sequence of events in sample space S such that $A_1 \subseteq A_2 \subseteq .... \subseteq A_n$ then prove that $P\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n \to \infty} P(A_n)$.

29. Establish weak law of large numbers for a random sample $X_1, X_2, ..., X_n$ drawn from a population with mean μ and variance σ^2.

30. Let $\{X_k\}$ be a sequence of independent random variables with values -2^k, 0 and 2^k and probabilities $P(X_k = \pm 2^k) = 2^{-(2k+1)}$; $P(X_k = 0) = 1 - 2^{-2k}$. Examine whether weak law of large numbers holds for the sequence.

31. Let the probability density function of a random variable X be $f(x) = 1;\ 0 < x < 1$. What is the lower bound of $P\left(\left|X - \frac{1}{2}\right| \leq 2\sqrt{\frac{1}{12}}\right)$ when one uses the Chebyshev's inequality?

32. Let $X_1, X_2, ..., X_n$ be a random sample from a uniform distribution over (0,1). Find the probability density function of r^{th} order statistic $X_{(r)}$.

33. Let $\overline{X}$ be the sample mean of a random sample of size 50 from a normal population with mean 112 and standard deviation 40. Find (a) $P(110 < \overline{X} < 114)$ (b) $P(\overline{X} > 113)$

34. Let X_1, X_2, X_3, X_4 be a random sample from a normal distribution with variance equal to 9 and let $S^2 = \frac{1}{3}\sum_{i=1}^{4}(X_i - \overline{X})^2$. Find k such that $P(S^2 \leq k) = 0.05$.

35. Let (X_1, X_2) be a random sample from a distribution with density function $f(x) = e^{-x};\ x > 0$ Find the density function of $Y = \min(X_1, X_2)$.

36. Find the second central moment μ_2 of a t distribution with n degrees of freedom.

37. If $X \sim F(n, n)$ is a F variable with (n, n) degrees of freedom, find the median of the distribution of X.

38. Let $(X_1, X_2, ..., X_m)$ and $(Y_1, Y_2, ..., Y_n)$ be two independent random samples of sizes m and n respectively from a standard normal population $N(0,1)$. What is the sampling distribution of $W = \dfrac{n\sum_{i=1}^{m} X_i^2}{m\sum_{i=1}^{n} Y_i^2}$. Hence obtain mean of W.

(6 $\times$ 4 = 24 Marks)

SECTION – D

(Answer **any two** Questions. Each question carries **15** marks)

39. (a) If X is a continuous random variable with mean μ and variance σ^2, establish Chebyshev's inequality.

(b) If X is a random variable with $E(X) = 3$ and $E(X^2) = 13$, use Chebyshev's inequality to determine the lower bound for the probability $P(-2 < X < 8)$.

40. (a) State and prove Lindberg-Levy form of central limit theorem.

(b) If $X_1, X_2, ..., X_n$ is a sequence of Bernoulli random variables with probability success p, write down the central limit theorem result.

41. (a) Suppose the mean weight of school children's book bag is 1.74 kilograms with standard deviation 0.22. Find the probability that the mean weight of a sample of 300 book bags will exceed 1.7 kilograms.

(b) Suppose the mean number of days to germination of a variety of seed is 22 with standard deviation 2.3 days. Find the probability that the mean germination time of a sample of 160 seeds will be within 0.5 day of the population mean.

42. If $X_1, X_2, ..., X_n$ is a random sample drawn from a population with distribution function $F(x)$ find the distribution function of rth order statistic $X_{(r)}$. If random variables are continuous obtain probability density function of $X_{(r)}$.

43. If $\{X_i\}$ is a sequence of independent standard normal random variables, find moment generating function of $Y = \sum_{i=1}^{n} X_i^2$. Identify the distribution of Y and write down its probability density function.

44. (a) Define t, χ^2 and F statistics and give relationship between each of them.

(b) Obtain r^{th} arbitrary moment μ_r' of F distribution with (m, n) degrees of freedom.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **M – 1565**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course VI

ST 1542 – ESTIMATION

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

(Use of statistical table and scientific calculator are allowed)

SECTION– A

Answer **all** questions. Each carries **1** mark:

1. If $x_1, x_2, ..., x_n$ is a random sample from a population $p^x(1-p)^{n-x}$, f or $x = 0,1$ and $0 < p < 1$. find the sufficient statistic for p.

2. Define unbiasedness.

3. T_1 and T_2 are two unbiased estimators of a parameter θ. When we say T_1 is more efficient than T_2?

4. Define minimum variance bound estimator.

P.T.O.

5. If $X_i,\ i = 1, 2, \ldots n$ follows exponential distribution mean $\left(\frac{1}{\theta}\right)$ then find the moment estimator of θ.

6. If T_n is a consistent estimator of θ, find the consistent estimator of e^{θ}.

7. What is the relation between sufficient estimator and a maximum likelihood estimator?

8. Give a large sample property of estimators.

9. Define a linear parametric function.

10. Define BLUE.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each carries **2** marks.

11. Distinguish between point estimation and interval estimation.

12. For the geometric distribution $f(x;\theta) = \theta(1-\theta)^{x-1},\ x = 1,\ 2, \ldots, 0 < \theta < 1$. Obtain art unbiased estimator of $1/\theta$.

13. Let X follows location exponential distribution with *pdf* $f(x) = e^{-(x-\mu)}, x > \mu$. Then find an sufficient statistic for μ.

14. Obtain the mle of β in $f(x) = (\beta + 1)x^{\beta}; 0 < x < 1$.

15. If $x_1,\ x_2, \ldots, x_n$ is a random sample from a uniform distribution over $(0,\ \theta)$. Obtain the moment estimator for θ.

16. Define confidence interval and confidence coefficient.

17. The diameter of cylindrical rods is assumed to be normally distributed with a variance 0.04 cm. A sample of 25 rods has a mean diameter of 4.5 cm. Find 95 % confidence limits for population mean.

18. Obtain a sufficient estimator for σ^2 in the $N(0,\ \sigma^2)$ distribution.

19. Show by an example that MLE need not be unbiased.

20. Distinguish between MVBE and MVUE.

21. Two samples from two normal populations having equal variances of size 10 and 12 have means 12 and 10 variances 2 and 5 respectively. Find 95 % confidence limits for the difference between two population means.
22. What is meant by efficiency of an estimator?
23. State Neyman's condition for sufficiency.
24. Two samples from two normal populations having equal variances of size 10 and 12 have means 12 and 10 and variances 2 and 5 respectively. Find 95% confidence limits for the difference between population means.
25. Write down the assumptions on error terms in a general linear model.
26. Show by an example that unbiasedness is not unique.

($8 \times 2 = 16$ Marks)

SECTION – C

Answer any **six** questions. Each carries **4** marks.

27. If $x_1, x_2, ..., x_n$ is a random sample from a normal population with mean μ and variance 1. Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_j^2$ is an unbiased estimate of $\mu^2 + 1$.
28. For a Poisson distribution with parameter θ, show that $\frac{1}{\bar{x}}$ is consistent for $\frac{1}{\theta}$.
29. Find the MLE of p for a binomial population with $p\,d\,f$ $f(x) = NC_x p^x (1-p)^{N-x}$, where N is known.
30. If the $p\,d\,f$ $f(x)$ of a population is given by $f(x) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2}$, $-\infty < x < \infty$. Examine whether θ has a minimum variance estimator.
31. Find the sufficient statistics for Gamma distribution with parameter α and β.
32. Explain the method of maximum likelihood estimation (MLE) of parameters. Write any two properties.
33. Let X_1, X_2, X_3 be a random sample of size 3 from $N(\mu, \sigma^2)$. Find the efficiency of $\frac{x_1 + 2x_2 + x_3}{4}$ relative to $\frac{x_1 + x_2 + x_3}{3}$.
34. A sample poll of 100 voters in a given district indicated that 55% of them were in favor of a particular candidate. Find 95% and 99% confidence limits for the proportion.
35. Prove that in a Normal distribution, sample mean is a unbiased, consistent and sufficient estimator of population mean.

36. Find the MLE of population mean of Poisson distribution. Check whether the estimator is sufficient or not.

37. Explain Gauss-Markov set up.

38. Observations on un correlated random variables Y_1, Y_2, Y_3 with common variance σ^2 are available with $E(Y_1) = \theta_1 - \theta_2 + \theta_3$, $E(Y_2) = \theta_1$, $E(Y_3) = \theta_3 - \theta_2$. Check whether $\theta_3 - \theta_2$ is estimable.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each carries **15** marks.

39. State and prove Gauss Markov theorem.

40. What are the desirable properties to be satisfied by a good estimator? Give one example each of estimators possessing each of the desirable properties.

41. Derive the confidence interval for the difference of proportions.

42. Let $x_1, x_2, ...x_n$. be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimators for

    (a) μ when σ^2 is known

    (b) σ^2 when μ is known

    (c) μ and σ^2 when both are unknown.

43. The sample values from population with *pdf* $f(x) = (1+\theta)x^{\theta}$, $0 < x < 1$, $\theta > 0$ are given below:

    0.46, 0.38, 0.61, 0.82, 0.59, 0.53, 0.72, 0.44, 0.59, 0.60

    Find the estimate of θ by the method of moments and maximum likelihood estimate.

44. (a) State Cramer Rao inequality.

    (b) Let $X_1, X_2, ..., X_n$ be a random sample from a population with pdf $f(x) = \theta e^{-\theta x}, x > 0, \theta > 0$.

    Find Cramer Rao lower bound for the variance of the unbiased estimator of θ.

(2 $\times$ 15 = 30 Marks)

(Pages : 6) **M – 1566**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course VII

ST 1543 : TESTING OF HYPOTHESIS

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

(Use of Statistical table and Scientific Calculator are allowed)

SECTION – A

Answer **all** questions. **Each** carries **1** mark.

1. Define critical region.

2. What is a statistical test?

3. Define size of the test.

4. What is the degrees of freedom of χ^2 in case of 2 × 2 contingency table?

5. Define power function.

6. The mean difference between 9 paired observations is 15 and the standard deviation of differences is 5. Find the value of *t* statistic.

P.T.O.

7. Name the appropriate test to test $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$ when the population is large and S.D is known.

8. If there are 10 symbols of two types, equal in number, give the maximum possible number of runs.

9. Define empirical distribution function.

10. When the number of treatments is 2 in Kruskal Wallis test, the test reduces to ____________.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** carries **2** marks.

11. Distinguish between simple and composite hypothesis.

12. Define null and alternate hypothesis.

13. State the assumptions of small sample test for population mean.

14. What are the uses of chi-square tests?

15. Given the following eight sample values –4, –3, –3, 0, 3, 3, 4, 4. Find the value of student's *t* statistic for testing $H_0 : \mu = 0$.

16. A manufacturer claims that his items could not have a large variance. 18 of his items has a variance 0.033. Find the value of Chi square to test $H_0 : \sigma^2 = 1$.

17. Define uniformly most powerful test.

18. The standard deviation of a sample of size 15 from a normal population was found to be 7. Examine whether the hypothesis that the S.D. is 7.6 is acceptable.

19. State Neyman Pearson lemma.

20. Explain the procedure for testing the significance of correlation coefficient.

21. If the observed and theoretical cumulative distribution functions are,

Observed c.d.f : 0.038, 0.066, 0.093, 0.177, 0.288, 0.316, 0.371

Theoretical cdf : 0.036, 0.042, 0.129, 0.159, 0.243, 0.275, 0.238

Find the value of K – S statistic.

22. Following are the yields of maize in q/ha recorded from an experiment and arranged in ascending order with median M = 20,

15.4, 16.4, 17.3, 18.2, 19.2, 20.9, 22.7, 23.6, 24.5

Test $H_0 : M = 20$ *vs* $H_1 : M \neq 20$ at $\alpha = 0.05$.

23. How Wilcoxen signed rank test differ from sign test?

24. How to resolve the problem of zero difference in sign test?

25. In what situations do we use nonparametric tests?

26. Define run.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** carries **4** marks.

27. Explain the terms (i) errors of the first and second kind (ii) critical region (iii) power of the test.

28. If $X \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against $H_1 : \theta = 1$ on the basis of a single observation from $f(x; \theta) = \theta e^{-\theta x}$, $x \geq 0$, obtain the probabilities of type 1 and type 2 errors.

29. A sample of 25 items were taken from a population with standard deviation 10 and the sample mean is found to be 65. Can it be regarded as a sample from a normal population with μ = 60.

30. How is the degrees of freedom of the Chi square for goodness of fit determined?

31. In tossing of a coin, let the probability of turning up a head *p*. The hypothesis are $H_0 : p = 0.4$ against $H_1 : p = 0.6$. H_0 is rejected if there are 5 or more heads in six tosses. Find the significance level of the test.

32. Suppose a random sample of size *n* is taken from the Poisson population with p.d.f

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \; x = 0, 1, 2, \ldots$$

Give the most powerful critical region of size α for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (\lambda_1 > \lambda_0)$.

33. It is claimed that more IAS selections are made from cities rather than rural places. On the basis of the following data do you uphold the claim?

	Selected	Not Selected
From Cities	500	200
From rural places	100	30

34. Explain likelihood ratio test.

35. Distinguish between large sample and small sample tests with examples.

36. Following is a sequence of heads (H) and tails (T) in tossing of a coin 14 times. HTTHHHTHTTHHTH

Test whether the heads and tails occur in random order.

[Given : for α = 0.05, r_L = 2, r_U = 12]

37. Explain Median test.

38. Explain Kolmogrov — Smirnov test.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** carries **15** marks.

39. (a) Explain the large sample test for testing equality of two population means.

(b) Given in the usual notation :

$$n_1 = 400,\ \bar{x}_1 = 250,\ s_1 = 40$$
$$n_2 = 400,\ \bar{x}_2 = 220,\ s_1 = 55$$

Test whether the two samples have come from populations having the same mean.

40. (a) Explain how the Chi square distribution may be used to test goodness of fit.

(b) Five dice were thrown 96 times and the number of times, at least one die showed an even number is given below :

No. of dice showing even number :	5	4	3	2	1	0
Frequency :	7	19	35	24	8	3

41. (a) Explain how *t* test is used for paired comparison of differences of means.

(b) The following data gives marks obtained by a sample of 10 students before and after a period of training. Assuming normality test whether the training was of any use.

Student No. :	1	2	3	4	5	6	7	8	9	10
Before :	91	95	81	83	76	88	89	97	88	92
After :	79	101	85	88	81	92	90	99	97	87

42. (a) Explain F test for equality of population variances.

(b) Two random samples drawn from two normal populations are :

Sample I :	20	16	26	27	23	22	18	24	25	19		
Sample II :	27	33	42	35	32	34	38	28	41	43	30	37

Obtain estimates of the variances of the populations and test whether the two populations have the same variance.

43. Explain Ordinary sign test and Wilcoxon signed rank test.

44. Explain Mann Whitney test and Kruskal Wallis test.

($2 \times 15 = 30$ Marks)

(Pages : 4) **M – 1567**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course VIII

ST 1544 : SAMPLE SURVEY METHODS

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define a statistical population.

2. Define probability sampling.

3. Give the expression of 100(1- α)% confidence interval of population mean for moderate sample size.

4. Define ‘inflation factor’ in sampling theory.

5. When will the design of a stratified sampling be preffered to that of SN?

6. Define ‘stratum weight’ in stratified sampling.

7. Define systematic sampling.

8. Give one advantage of systematic sampling.

P.T.O.

9. Define ratio estimator of population total.

10. Define linear regression estimator of population mean.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. What is sampling error?

12. Define mean square error of an estimator.

13. Define a sampling design.

14. Define a statistic, and give an example.

15. What is finite population correction?

16. What is an unbiased estimator?

17. What is the probability of selecting a random sample of size $'n'$ from $'N'$ units, without replacement?

18. Define the estimator of population mean in stratified sampling.

19. Explain proportional allocation.

20. Give an example where stratified sampling is suitable.

21. Give the difference between systematic and stratified sampling.

22. Give a systematic sample by Lahiri's method if $'N' = 23$ and $'n' = 5$.

23. Show that $\bar{y}_{sy}$ is unbiased for population mean when $N = nk$.

24. When will the ratio and regression estimates of population mean be the same?

25. Give an example of a ratio estimator for population mean.

26. Show that in simple random sampling the linear regression estimate $\bar{y}_{tr} = \bar{y} + b_0\left(\bar{X} - \bar{x}\right)$ is unbiased.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

27. Describe two differences between standard sample survey theory and classical sampling theory.

28. Give three uses of sample surveys.

29. Explain how the accuracy of an estimator is evaluated through confidence intervals.

30. Show that sample mean is unbiased in SRSWR.

31. Show that $V\left(\bar{y}_{WOR}\right) = \frac{N-n}{N}\frac{S^2}{n}$ in SRSWOR.

32. Compare the efficiency of sample mean under SRSWR and SRSWOR.

33. Obtain the expression of $V_{prop}\left(\bar{y}_{st}\right)$ in stratified sampling.

34. Establish an unbiased estimate of $V\left(\bar{y}_{st}\right)$ in stratified sampling.

35. Give the systematic samples when $'k'$ samples each of size $'n'$ are to be taken from $N = nk$ units denoted as $y_1, y_2, ...y_k, y_{k+1}, y_{k+2}, ...y_{2k}, ... y_{(n-1)k+1}, y_{(n-1)k+2}, ...y_{nk}$.

36. Obtain the expression of the variance of a systematic sample of size $'n'$ for estimating population mean when linear trend is there and $N = nk$.

37. Show that the leading term in the bias of ratio estimate is

$$E\left(\hat{R} - R\right) = \frac{1 - f}{n\overline{X}^2}\left(R\,S_x^2 - \rho\,S_y\,S_x\right).$$

38. Give the expression of $100\left(1 - \alpha\right)\%$ confidence interval of population total using ratio estimator using large samples, and explain the terms contained therein.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. Explain in detail the principal steps in a sample survey.

40. Show that sample variance is unbiased for σ^2 in SRSWR.

41. Explain how the value of sample size is decided in stratified sampling when cost is to be minimised for a specified variance.

42. Show that $V\left(\overline{y}_{st}\right) = \dfrac{\left(\sum W_h\,S_h\right)^2}{n} - \dfrac{\sum W_h\,S_h^2}{N}$ under Neyman allocation.

43. Show that systematic sampling is more precise than simple random sampling if the variance within the systematic samples is larger than the population variance.

44. Show that in simple random sampling the linear regression estimate $\overline{y}_{tr} = \overline{y} + b_0\left(\overline{X} - \overline{x}\right)$ has minimum variance when $b_0 = \dfrac{S_{yx}}{S_{x^2}}$, and $V_{\min}\left(\overline{y}_{lr}\right) = \dfrac{1 - f}{n}S_y^2\left(1 - \rho^2\right)$.

(2 $\times$ 15 = 30 Marks)

(Pages : 6) **P – 2590**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Statistics

Core Course

ST 1541 : PROBABILITY AND DISTRIBUTION III

(2013-2017 Admission)

Time : 3 Hours Max. Marks : 80

Instructions: Use of Scientific Calculators and Statistical tables are allowed.

PART – A

Answer **all** questions. **Each** question carries **1** mark.

1. Examine the existence of the $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} x^3 - 2 & \text{if } x \geq 2 \\ 2 + x^2 & \text{if } x < 2 \end{cases}$.

2. Suppose $f(x) = \begin{cases} \dfrac{2.\sin x}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ examine the continuity of $f(x)$ at x = 0.

3. Define absolute continuity of a function $f(x)$ at a point c.

4. State Rolle's theorem.

5. If $f(x) = e^x$, write the Maclaurin's series of $f(x)$.

P.T.O.

6. If $X_1, X_2, \ldots X_n, \ldots$ is a sequence of random variables explain the concept of Convergence in probability.

7. Using Chebyshev's inequality find the lower limit of $P\{|X-\mu| \le 2\}$ where X is a random variable with mean μ and variance 1.

8. Write down the probability density function of a random variable that follows Beta distribution of first kind with parameters m=1 and n=1.

9. If $X_1, X_2, \ldots X_n$ is a random sample from a normal population $N(0, 1)$ give the expression for χ^2 statistic with n degrees of freedom.

10. If x follows t distribution with n degrees of freedom, what is the mean of X?

(10 $\times$ 1 = 10 Marks)

PART – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. If $\lim_{x \to c} f(x) = L$ prove that $\lim_{x \to c} |f|(x) = |L|$.

12. Suppose that $f(x) = \begin{cases} A - x & \text{if } x < 2 \\ x^2 - 1 & \text{if } x \ge 2 \end{cases}$ where A is a constant. Find the value of A such that the function $f(x)$ is continuous at the point $x = 2$.

13. Explain the concept of removable discontinuity of a function $f(x)$ at a point c. Give an example.

14. Let $f(x)$ be a function defined over (a, b), explain left-hand and right-hand derivatives of $f(x)$ at a point $c \in (a,b)$.

15. State Lagrange's Mean Value theorem and give its geometric interpretation.

16. Find the mean of Beta distribution of second kind with parameters (m, n).

17. Let (X, Y) be a bivariate normal random variable with parameters $\mu_1 = \mu_2 = 0$, $\sigma_1 = 1, \sigma_2 = 1$ and $\rho = 1/2$, then write down the moment generating function of (X, Y).

18. Write a short note on Sampling distribution.

19. If $X_1, X_2, ...X_n$ is a random sample drawn from a population with moment generating function $M_X(t)$, show that $M_{\overline{X}}(t) = (M_X(t/n))^n$ where $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

20. If $t \sim t_{(6)}$ is a student's t variable with 6 degrees of freedom, using statistical table, find k such that $P(|t| \leq k) = 0.90$.

21. Let X and Y are two independent Chi-square random variables such that $X \sim \chi^2_{(3)}$ and $Y \sim \chi^2_{(3)}$. Identify the distribution of $X + Y$ and hence obtain mean of $X + Y$.

22. If $U \sim \chi^2_{(m)}$ and $V \sim \chi^2_{(n)}$ are two independent Chi-square random variables, what is the distribution of $X = \frac{U/m}{V/n}$?. Hence write down the value of mean of X.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. **Each** question carries **4** marks.

23. If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$,then prove that $\lim_{x \to c} (f + g)(x) = L + M$.

24. Verify whether the function $f(x) = \sin x$, where $x \in [0, \pi]$ satisfies the conditions of Rolle's theorem. Also find $c \in (0, \pi)$ such that $f'(c) = 0$.

25. If $X_1, X_2, ...X_n$ is a sequence of Bernoulli random variables with probability success ρ, write down the central limit theorem result.

26. Let the probability density function of a random variable X be $f(x) = \frac{1}{2} : -1 < x < 1$.

Find the lower bound of $P\left(|X| \le 2\sqrt{\frac{1}{2}}\right)$ using Chebyshev's inequality.

27. Let $X_1, X_2, ...X_n$ be the observations from a random sample of size n from a normal distribution with mean μ and variance σ^2. Derive the expression for mean and variance of the sample mean $\overline{X}$.

28. Find the r^{th} arbitrary moment $\mu_r^{'}$, of log normal distribution with parameters μ and σ^2. Hence obtain mean of the distribution.

29. If $\chi^2 \sim \chi^2_{(n)}$ where n > 2, obtain the point at which the probability density function of χ^2 attains maximum.

30. If (X_1, X_2, X_3, X_4) is a random sample of size 4 from a standard normal population N(0, 1), write down the sampling distribution of following statistics.

(i) $U = \frac{\sqrt{3}X_4}{\sqrt{X_1^2 + X_2^2 + X_3^2}}$

(ii) $V = \frac{X_1^2 + X_2^2}{X_3^2 + X_4^2}$

31. Derive the relationship between t and F distribution.

(6 $\times$ 4 = 24 Marks)

PART – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (i) Show that differentiable functions are continuous. Is the converse true? Justify your answer.

    (ii) Show that every monotonic function is Riemann integrable.

    (iii) Let $f(x)$ be a function defined on [0, 1] as follows:

$$f(x) = \begin{cases} 1 & \text{when x is rational} \\ 0 & \text{when x is irrational} \end{cases}$$

    Examine whether $f(x)$ is Riemann Integrable on [0, 1].

33. (i) State Lindberg-Levy form of Central limit theorem.

    (ii) State and prove weak law of large numbers.

    (iii) Let $X_1, X_2, ...X_n$ be a random sample of size n = 16 drawn from a Poisson distribution with mean λ=2. Use central limit theorem to find $P(1 \leq \overline{X} \leq 3)$ where $\overline{X} = \dfrac{X_1 + X_2 + ... + X_n}{n}$.

34. (i) Define Cauchy distribution and state its significance.

    (ii) If X follows beta distribution of first kind with parameters (m, n) find the first two arbitrary moments of X.

    (iii) Obtain mean of F distribution with (m, n) degrees of freedom.

35. (i) Find the moment generating function of Gamma distribution with one parameter α. Hence obtain mean and variance of Gamma distribution.

(ii) If random variables $X_1, X_2, ... X_n$ are independent and identically distributed standard normal random variables, find the moment generating function of $U = \sum_{i=1}^{n} X_i^2$ and hence identify the distribution of U.

(2 $\times$ 15 = 30 Marks)

(Pages : 6) **P – 2591**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Statistics

Core Course

ST 1542 : ESTIMATION

(2013-2017 Admissions)

Time : 3 Hours Max. Marks : 80

Instructions: Use of Scientific Calculators and Statistical tables are allowed.

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. What do you mean by Point estimator of a parameter?

2. Define unbiasedness of an estimator.

3. State sufficient conditions for the consistency of an estimator.

4. If T is an unbiased estimator of parameter θ, what is the value of Mean squared error of T?

5. State factorization theorem on sufficiency of an estimator.

P.T.O.

6. If $X_1, X_2, ...X_n, ...$ is a random sample drawn from a population with probability density function $f(x;\theta) = \theta . e^{-\theta x}; x \geq 0; \theta > 0$, write down the likelihood function.

7. Give a note on minimum variance unbiased estimator.

8. Write the confidence interval for the mean of normal distribution when population standard deviation σ is known.

9. Define quadratic loss function.

10. Define Bayes risk of a decision.

(10 $\times$ 1 = 10 Marks)

PART – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Define Statistic and give one example.

12. Let $X_1, X_2, ...X_n$ be a random sample drawn from a population with mean μ. Show that sample mean $\overline{X}$ is an unbiased estimator of population mean μ.

13. Let $(X_1, X_2, ...X_n)$ be a random sample from a population with probability density function $f(x;\theta) = \frac{1}{\theta}; 0 < x < \theta$; $\theta > 0$. If $\overline{X}$ denotes the sample mean, what should be the value of K such that $K\,\overline{X}$ is an unbiased estimator of θ?

14. Let $X_1, X_2, ...X_n, ...$ be independent and identically distributed Bernoulli random variables with parameter p then show that $\frac{1}{n}\sum_{i=1}^{n} X_i$ is consistent for p.

15. If X_1, X_2, X_3 are three independent observations from a population with mean μ and variance σ^2 and if $\tau_1 = X_1 + X_2 - X_3$ and $\tau_2 = 2X_1 + 3X_2 - 4X_3$ are two estimators of μ, compare the efficiencies of τ_1 and τ_2.

16. State Cramer-Rao inequality.

17. Let $X_1, X_2, ...X_n, ...$ be a random sample from the population with probability density function $f(x;\theta) = \theta.x^{\theta-1};\ \ 0 < x < 1$. Show that $Y = X_1, X_2, ...X_n$, is a sufficient estimator of θ.

18. Give two properties of Maximum likelihood estimator.

19. Let $(X_1, X_2, ...X_n)$ be a random sample drawn from a population with probability density function $f(x) = \dfrac{1}{b-a}, a \le x \le b$. Obtain Maximum Likelihood Estimator of a and b.

20. Explain method of least squares to find estimate of a parameter.

21. What do you mean by admissible and inadmissible decision rule?

22. Explain Risk function in decision theory.

(8 $\times$ 2 = 16 Marks)

PART – C

Answer any **six** questions. **Each** question carries **4** marks.

23. Let $(X_1, X_2, ...X_n)$ be a random sample drawn from a population with mean μ and variance σ^2, show that $\sum_{i=1}^{n} a_i.X_i$ is an unbiased estimator of μ for any set of values $(a_1, a_2, ..., a_n)$ satisfying $\sum_{i=1}^{n} a_i = 1$.

24. Show that an unbiased estimator T of θ whose variance tends to zero as the sample Size increases is a consistent estimator of θ.

25. For a Poisson distribution with parameter λ, show that $\frac{1}{\overline{X}}$ is consistent estimator of $\frac{1}{\lambda}$, where $\overline{X}$ is the sample mean of the sample drawn from the population.

26. If $X_1, X_2, ...X_n, ...$ is a random sample drawn from an exponential distribution with parameter $f(x:\theta) = \frac{1}{\theta}.e^{-(x/\theta)}$; $x > 0$; $\theta > 0$, find the Cramer-Rao lower bound for the variance of the unbiased estimator of θ.

27. Obtain the maximum likelihood estimator of θ using a random sample $X_1, X_2, ...X_n$ drawn from the distribution with probability density function $f(x:\alpha) = e^{-(x-\alpha)}$, $x \geq \alpha; \alpha > 0$.

28. If $x_1 = 0.2, x_2 = 0.4, x_3 = 0.5, x_4 = 0.9$ are values of a random sample of size 4 from a population with probability density function $f(x;\theta) = \frac{3x^2}{\theta^3}, 0 \leq x \leq \theta$, what is the estimate of θ using method of moments.

29. Let $(X_1, X_2, ...X_n)$ be a random sample from a population with probability density function $f(x:\theta) = \frac{1}{\theta}e^{-x/\theta}$; $x \geq 0$; $\theta > 0$. Find the estimator of θ using method of moments.

30. A survey was conducted in a college before the election to Students' union. Of 80 students contacted, 32 said they would vote for candidate A. Obtain 95% confidence limits for the proportion of voters in the population in favour of Candidate A.

31. Explain the procedure of construction of $(1-\alpha)100\%$ confidence interval for μ, where μ is the population mean, using a small sample drawn from the normal population with unknown population variance.

(6 $\times$ 4 = 24 Marks)

PART – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (i) Show that sample variance $s^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_1 - \overline{X}\right)^2$ of a random sample $\left(X_1, X_2, ...X_n\right)$ drawn from a population with variance σ^2, is a biased estimator of σ^2. Hence deduce an expression of an unbiased estimator of population variance σ^2.

(ii) If T is a consistent estimator of θ, then prove that T^2 is a consistent estimator of θ^2.

33. (i) Explain the method of maximum likelihood estimation of a parameter θ.

(ii) Let $\left(X_1, X_2, ...X_n\right)$ be a random sample from a normal population $N\left(0, \sigma^2\right)$ where σ^2 is unknown. Find the maximum likelihood estimator of σ^2.

34. (i) Find a sufficient estimator for the parameter θ of a uniform distribution defined on (0, θ).

(ii) Let $\left(X_1, X_2, ...X_n\right)$ be a random sample from a Bernoulli distribution with probability mass function $f(x : p) = p^x (1-p)^{1-x}$; $x = 0, 1$; $0 < p < 1$. Verify the sufficiency of the estimator $T = \sum_{i=1}^{n} X_i$, with respect to the estimator p.

35. (i) Obtain $(1-\alpha)$100% confidence interval for σ^2 of a normal population with mean μ and variance σ^2.

(ii) Derive the confidence interval for the proportion of a population using a large sample drawn from the population.

(iii) Of the 250 patients of certain disease treated with certain drug, 180 are cured. Set 95% confidence interval of population proportion of patients likely to be cured in future by administering the said drug.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **P – 2592**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme Under CBCSS

Statistics

Core Course

ST 1543 : TESTING OF HYPOTHESIS

(2013-2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Differentiate hypothesis and statistical hypothesis.
2. Explain the power of the test.
3. Define composite hypothesis.
4. Define critical region.
5. What is the form of the likelihood ratio test statistic?
6. How do you apply likelihood ratio test in large samples?
7. Define large sample test.
8. Give an example of a statistic which follows students t-distribution.

9. Define empirical distribution function.

10. What you mean by kernal?

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Explain the steps in statistical testing procedure.

12. Explain level of significance and critical region.

13. Define the most powerful test with fixed size α.

14. Discuss the relation between simple likelihood ratio and the test given by Neyman and Pearson.

15. Discuss the advantages of likelihood ratio (LR) best.

16. Define χ^2 test for goodness of fit.

17. Discuss the construction of a test statistic for testing the population proportion has a specified value.

18. Give two test statistics which follows student's t-distribution.

19. What are the assumptions of paired t-test?

20. Define U-statistics with an example.

21. Define estimable parametric function.

22. Define one sample sign test.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. **Each** question carries **4** marks.

23. Discuss the two types of errors which creeps in the statistical testing problem. Give the general idea of managing these errors.

24. Find the size and power of the following test. Reject $H_0 = \lambda = 1$ in favour of $H_1 : \lambda = 2$ whenever $X_1 + X_2 \geq 2$. Assume that X_1 and X_2 are independent observations from Poisson (λ) distribution.

25. Explain the Neymam Pearson approach for deriving the most powerful test.

26. Discuss the way of testing equality of variances of two normal populations.

27. A sample of 25 items were taken from a population with standard deviation 10 and the sample mean is found to he 65. Can it be regarded as a sample from a normal population with μ = 60.

28. Explain chi-squre test for independence of attributes.

29. Discuss the procedure of testing the significance of correlation coefficient.

30. Explain median test for two sample problem.

31. Distinguish between one sample and two sample run test.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. **Each** question carries **15** marks.

32. (a) The hypothesis $H_0 : \theta = 2$ is accepted against $H_1 : \theta = 5$ if $X \leq 3$ when X has an exponential distribution with mean θ. Find the type I and type II error probabilities of the test.

    (b) Let X follows $B(10, \rho)$. Consider the following test for testing $H_0 : \rho = 1/2$ against $H_1 : \rho = 1/4$. Reject H_0 if $X \leq 2$. Find the significance level and power of the test.

33. Suppose X follows $N(\mu, 1)$. Derive the MP test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$ based on sample size n.

34. (a) Develop the test procedure of testing equality of variances of two normal populations.

    (b) Explain chi-square test for independence of attributes.

35. Describe Kolmogorov-Smirnov one sample and two sample problems.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **P – 2593**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Statistics

Core Course VIII

ST 1544 : SAMPLE SURVEY METHODS

(2013-2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION A

Answer **all** questions. Each question carries **1** mark.

1. Write the ratio estimate of population total.

2. What is the probability of selecting a simple random sample of size n from a population having size N under without replacement?

3. Define sample.

4. Explain the basic principle of sampling

5. What is sampling frame?

6. What are the two important errors in sample survey?

7. What are the points which need special attention in the preparation of sampling frame?

8. What are strata?

P.T.O.

9. Explain any one method of selecting a simple random sample without replacement.

10. Write any two merits of systematic sampling.

($10 \times 1 = 10$ Marks)

SECTION B

Answer any **eight** questions. Each question carries **2** marks.

11. Describe census method with an example.

12. What are the advantages of sample surveys over census survey?

13. Distinguish between probability and non probability sampling.

14: How does sampling without replacement differ from sampling with replacement?

15. What is optimum allocation in stratified sampling?

16. Describe circular systematic sampling.

17. What are the uses of random number table?

18. Write the variance of sample mean $\bar{y}$ in systematic sampling.

19. Find the regression estimate of population mean.

20. Show that $V(\bar{y}_{st})_{prop} \leq V(\bar{y})_{srs}$.

21. Find the gain in precision under stratified sampling over simple random sampling.

22. Find the relative efficiency of the estimate of population mean in systematic sampling over SRSWOR.

($8 \times 2 = 16$ Marks)

SECTION C

Answer any **six** questions. Each question carries **4** marks.

23. Write any two ways of allocating the sample size in different strata in stratified sampling method.

24. What is systematic sampling? Find $V(\bar{y}_{sys})$.

25. Show that SRSWOR, the sample mean $\bar{y}$ is an unbiased estimator $\bar{Y}$. Also identify its sampling variance.

26. Explain the procedure of selecting a random sample under stratified random sampling method.

27. Compare systematic sampling verses stratified random sampling.

28. Explain the ratio method of estimation.

29. Describe briefly the concept of regression method of estimation.

30. Suggest an unbiased estimate of population mean $\bar{Y}$ under stratified sampling and obtain the expression for its variance under Neyman allocation.

31. Find V ($\bar{y}$) under SRSWR.

(6×4 = 24 Marks)

SECTION D

Answer any **two** questions. Each question carries **15** marks.

32. Show that $V(\bar{y}_{st})_{opt} \leq V(\bar{y}_{st})_{prop} \leq V(\bar{y})_{srs}$.

33. What is the ratio estimate of population variance? Derive the bias of the population variance.

34. Show that $V\left(\overline{y}_{sys}\right)=\frac{nk-1}{nk}\frac{S^2}{n}\left(1+(n-1)\rho\right)$, where ρ is the interclass correlation coefficient.

35. Explain :

    (a) Sampling Design

    (b) Mixed sampling

    (c) Judgement sampling

    (d) Non-sampling error.

(2 $\times$ 15 = 30 Marks)

(Pages : 6) **P – 2598**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Statistics

Core Course V

ST 1541 – LIMIT THEOREMS AND SAMPLING DISTRIBUTIONS

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

Statistical Table is permitted

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define sigma field.
2. If $A \subseteq B$ prove $P(A) \leq P(B)$, using axioms of Probability.
3. Define convergence in probability.
4. State Weak law of large numbers.
5. Define Chi square distribution with n degrees of freedom.
6. What is sampling distribution?
7. Define non central t distribution.
8. What is the relation between Student's t and F distribution?

P.T.O.

9. Define rth order statistic $X_{(r)}$.

10. If X_1, X_2, ... X_n be a random sample from $U(0, \theta)$, find the distribution function of $Max(X_1, X_2 ..., X_n)$.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks

11. Define limit supremum and limit infimum for a sequence of events $\{A_n\}$.

12. Suppose $\{X_n\}$ is a sequence of random variables with probability mass function $P(X_n = 1) = \frac{1}{n}$ and $P(X_n = 0) = 1 - \frac{1}{n}$. Examine the convergence of $\{X_n\}$ in probability.

13. State Borel – Cantelli lemma.

14. What are the assumptions on Central Limit theorem?

15. A random variable X has mean 50 and variance 100. Use Chebychev's inequality to obtain the upper bound for $P\{|X - 50| \geq 15\}$.

16. List the advantages of Chebychev's inquality.

17. Let $X_1, X_2 ...., X_{100}$ be a random sample of size 100 drawn from a population with mean 10 and variance 9, then find $P(\overline{X} > 10.5)$ using central limit theorem.

18. Distinguish between parameter and statistic.

19. If the sample values are 1, 3, 5, 6, 9, find the standard error of the sample mean.

20. If $X_1, X_2, ..., X_n$ are independent $N(\mu, \sigma^2)$ random variables, find the distribution of $\sum_{i=1}^{n} \left(\frac{x_1 - \mu}{\sigma} \right)^2$.

21. Find the mode of the Chi-square distribution with n degrees of freedom.

22. What are the characteristics of t distribution?

23. What are the properties of non central F distribution?

24. Let X and Y be independent standard normal variates. State the distribution of $\frac{x^2}{y^2}$ and write down the pdf.

25. Let $(X_1, X_2 ... X_n)$ be a random sample of size n from a distribution with density function f(x) and distribution function F(x). Write down the distribution function and probability density function of the nth order statistic, $X_{(n)}$.

26. Le $(X_1, X_2 ... X_n)$ be a sequence of independent normal random variables with pdf $f(x) = e^{-(x-\theta)}, x > \theta$. Find the pdf of $Min(X_1, X_2 ... X_n)$.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

27. If $A_1, A_2, ..., A_n ...$ is a sequence of events in sample space S such that $A_1 \subseteq A_2 \subseteq ... \subseteq A_n ...$ then prove that $P\left(\bigcup_{n=1}^{\infty} A_n \right) = \lim_{n \to \infty} P(A_n)$.

28. Show by an example that a sequence of distribution functions need not always converge to a distribution function.

29. State and prove Bernoulli's law of large numbers.

30. Examine whether the weak law of large numbers holds good for the sequence $\{X_n\}$ of independent random variables where $P\left\{X_n = \frac{1}{\sqrt{n}}\right\} = \frac{2}{3}, P\{X_n = -\frac{1}{\sqrt{n}}\} = \frac{1}{3}$.

31. A random sample of size 16 is taken from a normal population with mean 30 and variance 64. Find the probability that the sample variance s^2 will be less than the population variance.

32. Give the properties of Chi square distribution and examine its relationship with the normal distribution.

33. What are the mean and variance of sample variance s^2?

34. If X_1, X_2, X_3 and X_4 are independent observations from a univariate normal with mean zero and unit variance. Find the distribution of (a) $U = \frac{\sqrt{2}\, x_3}{\sqrt{x_1^2 + x_2^2}}$ (b) $V = \frac{3x_4^2}{x_1^2 + x_2^2 + x_3^2}$.

35. If two independent random samples of sizes 15 and 20 are taken from $N(\mu, \sigma)$, what is $P\left(\frac{s_1^2}{s_2^2} < 2\right)$?

36. Give two examples of a statistic following Students *t* distribution.

37. Using Statistical table, find the right tailed critical value corresponding to area 0.05 for (a) Chi-square distribution with 15 degrees of freedom (b) *t* distribution with 16 degrees of freedom.

38. Let X1, X2, ... ,X_n be n independent variates, X_i having Geometric distribution with parameter $p_i, i.e., P(X_i = x_i) = q_i^{x_i - 1} p_i, q_i = 1 - p_i, x_i = 1, 2, 3, \ldots$ Show that $X_{(1)}$ is distributed geometrically with parameter $(1 - q_1 q_2 \ldots q_n)$.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. (a) State and prove Lindberg-Levy form of central limit theorem.

(b) Show by using central limit theorem that if X follows the binomial distribution with parameters (n, p), its distribution will tend to normal as $n \to \infty$.

40. (a) State and prove Chebychev's inequality.

(b) For the geometric distribution $f(x) = 2^{-x}, x = 1, 2, 3\ldots$ prove that Chebychev's inequality gives $P\{|X - 2| \leq 2\} > 1/2$, while the actual probability is 15/16.

41. (a) Derive the sampling distribution of means of samples chosen from a normal population.

(b) A population is known to follow normal distribution with mean 2 and S.D. 3. Find the probability that the mean of a sample size 16 taken from this population will be greater than 2.5.

42. (a) Derive the moment generating function of Chi square distribution with n degrees of freedom and hence obtain its mean and variance.

(b) State and prove the additive property of Chi square distribution.

43. (a) Define t, χ^2 and F statistics and establish relationship between each of them.

(b) If F (m, n) represents a F variate with (m, n) degrees of freedom, prove that $P[F(m,n) \geq c] = \left[F(n,m) \leq \frac{1}{c}.\right]$

44. Let X_1, X_2, ... ,X_n be a random sample from a population with cumulative density. Show that Y_1 = Min (X_1,X2, ... ,X_n,) is exponential with parameter $n\lambda$ if and only if each X_i is exponential with parameter λ.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **P – 2599**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Statistics

Core Course VI

ST 1542 – ESTIMATION

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION A

Answer **all** questions. Each question carries **1** mark.

1. Define estimator.
2. Give an example of unbiased estimator.
3. Give the large sample property of a good estimator.
4. Define parametric space.
5. Give an example of a sufficient statistics.
6. Define minimum variance unbiased estimator.
7. Describe linear parametric function.
8. Define confidence coefficient.
9. Define likelihood function.
10. Define maximum likelihood estimator.

(10 $\times$ 1 = 10 Marks)

P.T.O.

SECTION B

Answer any **eight** questions. Each question carries **2** marks.

11. What do you understand by point estimation?

12. If T is an unbiased estimator of θ, examine whether T^2 is unbiased estimator of θ^2.

13. State Neymann's factorization theorem and mention its uses.

14. Show that in Cauchy distribution.

    $f(x,\theta)=\frac{1}{\pi\left[1+(x-\theta)^2\right]}, -\infty < x < \infty$

    Sample median is a consistent estimator of θ.

15. Find sufficient estimator of θ in $f(x,\theta)=\theta e^{-\theta x}, x>0, \theta>0$ if it exist.

16. What is sampling distribution?

17. Find the maximum likelihood estimator of λ in $P(\lambda)$.

18. Explain the method of construction of confidence interval.

19. Describe linear estimation.

20. Give any two important properties of moment estimator.

21. Find 95% confidence interval of population proportion.

22. Describe minimum variance bound estimator.

23. Describe the method of least squares.

24. Explain estimability.

25. Find the consistent estimator of population mean in normal population.

26. If $(x_1, x_2 ......x_n)$ is a random sample drawn from $N(\mu, 1)$ show that $t=1/n \ \xi^n \ x^2$ is unbiased estimator of μ^2+1.

(8 $\times$ 2 = 16 Marks)

SECTION C

Answer any **six** questions. Each question carries 4 marks.

27. State and prove the sufficient set of conditions for consistency of an estimator.

28. Let (X_1, X_2, X_3) be a random sample of size three drawn from a population with mean μ and variance σ^2. Let T_1, T_2 and T_3 be three estimators of μ. Where $T_1 = X_1 + X_2 - X_3$; $T_2 = 2X_1 + 3X_2 - 4X_3$; $T_3 = (X_1 + X_2 + X_3)/3$
    (a) Examine whether T_1, T_2 and T_3 are unbiased estimators of μ or not.
    (b) Which estimator is more efficient? Why?

29. Show that minimum variance unbiased estimator is unique.

30. Find a sufficient estimator of θ based on a random sample of size n drawn from uniform distribution with parameter θ.

31. A random sample $(x_1, x_2......x_n)$ is taken from $N(0, \sigma^2)$. Examine whether $t = 1/n$ ξ^n x_i^2 is a minimum variance bound estimator of σ^2.

32. Explain Gauss-Markov set-up.

33. Obtain 100 (1 – α)% confidence interval of σ^2 based on sample observations drawn from $N(\mu, \sigma^2)$.

34. A random sample of 500 apples was taken from a large consignment and 60 were found to be bad apples. Find 98% confidence interval for the proportion of bad apples in the consignment.

35. Explain the desirable properties of a good estimator.

36. Examine whether sample variance is an unbiased and consistent estimator of population variance in $N(\mu, \sigma^2)$.

37. Obtain the moment estimator of θ based on sample observations drawn from $f(x, \theta) = \theta e^{-\theta x}$, $x > 0, \theta > 0$.

38. State and establish the necessary and sufficient condition for estimability of a linear parametric function.

(6 $\times$ 4 = 24 Marks)

SECTION D

Answer any **two** questions. Each question carries **15** marks.

39. (a) Discuss the method of maximum likelihood estimation. Also describe any two important properties of M.L.E.

    (b) Obtain the maximum likelihood estimators of μ and σ^2 in $N(\mu, \sigma^2)$.

40. (a) Describe the method of moments. Examine whether moment estimators are

    (i) unbiased

    (ii) unique.

    (b) Find the moment estimators of α and β in

    $f(x, \alpha, \beta) = \frac{\beta^2}{\alpha} e^{-\beta x} X^{\alpha-1}, 0 < x < \infty$,

41. (a) Let $X_1, X_2, X_3, ...X_n$ be a random sample of size n drawn from a population with $f(x, \theta) = e^{-(x-\theta)}$, $0 < x < \infty$, $-\infty < \theta < \infty$

    Find sufficient estimator of θ.

    (b) Obtain the minimum variance bound estimator of μ in $N(\mu, \sigma^2)$ when σ^2 is known. Also find the variance of the estimator.

    (c) Find an unbiased estimator of θ in $f(x, \theta) = \theta(1-\theta)^{x-1}$, $x = 1, \infty........$, $0 < \theta < 1$.

42. (a) Obtain the 100 $(1-\alpha)$% confidence interval of $P_1 - P_2$, where P_1 and P_2 are proportions of two independent populations.

    (b) Show that for the distribution $f(x, \theta) = \theta e^{-\theta x}$, $x > 0$ the central confidence limits for large samples with 95% confidence coefficient are given by $\left(1 \pm \frac{1.96}{\sqrt{n}}\right) \Big/ \overline{x}$.

43. Find 100 $(1-\alpha)$% confidence interval of $\mu_1 - \mu_2$, where μ_1 and μ_2 are means of two independent normal populations when

    (a) the population variances are unknown and different.

    (b) when the population variances are common and unknown.

44. State and prove Gauss-Markov's theorem.

(2 × 15 = 30 Marks)

(Pages : 6) **P – 2600**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Statistics

Core Course VII

ST 1543 – TESTING OF HYPOTHESIS

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

(Statistical Table and Calculator and permitted)

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. What is a simple hypothesis?

2. What do you mean by best critical region?

3. Define the power of a test.

4. What do you mean by most powerful test?

5. What are likelihood ratio tests?

6. What are the conditions to be satisfied to apply a Chi-square test for goodness of fit.

P.T.O.

7. Which test statistics is used to test for a hypothetical value of the proportion of observations when a sample is taken from a Binomial population?

8. What are the assumptions of a t-test?

9. Define the empirical distribution function.

10. What is the indication of the number of runs in a run test?

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Distinguish between the level of the test α and the p value.

12. A population follows Normal distribution with parameters μ and σ = 3. To test the hypothesis $H_0 = \mu = 5$ *vs* $H_1 : \mu = 7,$ the test procedure suggested is to reject H_0 if $\overline{x} \geq 6$ where $\overline{x}$ is the mean of a sample of size 16. Find the significance level and power of the test.

13. What are the steps for carrying out a statistical test procedure?

14. Using the Neymann-Pearson Lemma, find the best critical region for the test $H_0 : \mu = \mu_0$ *vs* $H_1 : \mu = \mu_1,$ when μ_1 and σ^2 is known.

15. When do you call a test uniformly most powerful?

16. A coin is tossed 900 times in which head appears 490 times. Does it support the claim that the coin is unbiased?

17. Outline the large sample test for testing the specified variance of the population.

18. From a population with mean 200 and unknown SD, a sample is taken with mean 195 and S.D 50.If the null hypothesis is rejected at 5% level, what is the least sample size?

19. Discuss the applications of chi-square distribution.

20. Briefly discuss the test procedure of testing the equality means of the population with small samples.

21. The S.Ds of two samples of sizes 10 and 14 from two normal populations are 3.5 and 3.0 respectively. Examine whether the two populations have the same variances.

22. Define an F statistic. What are its uses?

23. In two Colleges affiliated to a University 40 out of 250 and 49 out of 200 students failed in an examination. If the overall percentage of failure in the University is 20, examine whether the two Colleges differ in performance significantly.

24. Discuss the runs test for randomness.

25. What is the procedure of a sign test for one sample?

26. Explain the Mann-Whitney U test. For which parametric test this is an alternative?

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

27. Let X ~N $(\mu, \sigma^2), \mu$ is the unknown mean and $\sigma^2 = 4$.To test $H_0 : \mu = -1 \; vs \; H_1 : \mu = 1$ based on a sample of size 10 from this population, we use the critical region $x_1 + 2x_2x + 3x_3 + \dots + 10x_{10} \geq 0$.What is its size? What is the power of the test?

28. State the Neyman-Pearson lemma and give its relevance.

29. Examine whether a Best Critical Region exists for testing $H_0 : \theta = \theta_0, vs\ H_1 : \theta > \theta_0$ for the parameter θ of $f(x;\theta) = \frac{1+\theta}{(x+\theta)^2}, 1 \leq x < \infty$.

30. Mention the properties of likelihood ratio test.

31. A sample of 400 observations were taken from a population with S.D 15. If the mean of the sample is 27, test whether the hypothesis that the population mean is greater than 24.

32. A manufacturer of dry cells claimed that the life of their cells is 24.0 hours. A sample of 10 cells had mean life of 22.5 hours with a S.D 3.0 hours. On the basis of this information test whether the claim is valid or not.

33. Write the procedure for carrying out a chi-square test of homogeneity.

34. Test whether the following figures provide the evidence of the effectiveness on inoculation.

	Attacked	Not attacked
Inoculated	120	80
Not Inoculated	180	420

35. Two samples of 6 and 5 items respectively gave the following data: mean of the first sample =40,S.D of the first sample =8, mean of the second sample = 50 S.D of the second sample =10. Is the difference in means significant at 5% level?

36. What are the advantages and disadvantages of non-parametric methods over parametric methods?

37. A public school official felt that high school sections of a large school system would tend to score higher on a standard reading examination than the national median of 50. He randomly select 13 students and gave them the test. The results are as follows:57,70,42,48,72,63.45,66,59,39,73,78,47. Is his feeling correct? Use Wilcoxon's signed rank test. (From tables with n=13, $\alpha = .01$ table value C=13).

38. Write a comparison between Chi-square test of goodness-of—fit and a Kolmogrov-Smirnov test.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. (a) Show that every most powerful critical region is unbiased.

(b) Given a random sample of size n from a population with pdf $f(x,\sigma) = \frac{1}{\sigma} e^{-\frac{x}{\sigma}}, x > 0, \sigma > 0$. Show that there exists no UMP test for testing $H_0 : \sigma = \sigma_0, vs\, H_1 : \sigma \neq \sigma_0$.

40. (a) Give the procedure for testing the equality of means of two normal populations when the population variances are known.

(b) Two independent samples from two Normal populations are shown below. Test whether the two populations have the same variance.

Sample I	60	65	71	74	76	82	85	87		
Sample II	61	66	67	85	78	63	85	86	88	91

41. (a) A sample of 27 pairs of observations from a Normal population gives the sample correlation coefficient *r* = 0.6. Is it likely that the variables are correlated?

(b) Discuss the test for equality of correlation coefficients of two populations.

42. (a) Explain the paired t test, mentioning the assumptions involved.

(b) An IQ test was administered to 5 persons before and after they were trained. The results are as follows:

Candidate :	A	B	C	D	E
IQ before training :	110	120	123	132	125
IQ after training :	120	118	125	136	121

Test whether there is any change in IQ after the training.

43. (a) Discuss the test for equality of means of two normal populations (i) with known S,Ds and (ii) with the same but unknown S.D.

(b) Two samples from normal populations gave the following results.

Sample size	Mean	S.D.
12	1050	68
10	980	74

Do the samples come from the same population with $\sigma_1^2 = \sigma_2^2$, unknown.

44. (a) Explain the Kolmogrov-Smirnov one sample test procedure.

(b) Discuss the procedure of the Wilcoxon's matched pair signed rank test.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **P – 2601**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Statistics

Core Course VIII

ST 1544 – SAMPLE SURVEY METHODS

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define sampling frame.

2. What are the objectives of sampling?

3. Mention some situations in which sampling methods alone can be adopted to collect data.

4. What is meant by purposive/subjective sampling?

5. Discuss the problem of non-response in a Sample Survey.

6. Name any two methods of selecting simple random samples.

7. Define finite population correction.

8. When do you go for stratification?

P.T.O.

9. Define a circular systematic sample.

10. Describe the use of auxiliary information in sampling.

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. What do you mean by Judgment sampling?

12. Distinguish between standard error and standard deviation.

13. How can one estimate the population total and its variance?

14. What do you mean by Mean Square Error? When do you use it?

15. Explain the errors in sampling.

16. How do you determine the sample size in SRSWOR.

17. Define a Simple Random Sample With Replacement (SRSWR).

18. Highlight the advantages of stratification.

19. What do you mean by proportional allocation in stratified sampling?

20. What do you mean by post stratification? Will it increase the precision of the estimate? Establish your claim.

21. What is balanced systematic sample.

22. What are the disadvantages of systematic sampling?

23. Distinguish between stratified sampling and cluster sampling.

24. What is meant by difference estimator?

25. What are the two types of ratio estimates in Stratified sampling?

26. What is meant by linear regression estimator for the population mean?

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

27. Define snowball sampling. In which situation it is used?

28. What are the advantages and disadvantages of sampling over census?

29. Show that in SRS $P_{ir} = \frac{1}{N}$, where P_{ir} denotes the probability of selecting the ith unit in the rth draw in a population of size N.

30. In SRSWR, show that an estimate based on d distinct units is superior to the one based on n units (d <n).

31. Establish that in SRSWOR, the sample mean square is an unbiased estimate of the population mean square.

32. Compare the efficiency of Systematic Sample with SRSWOR in terms of the intra class correlation coefficient.

33. Obtain the expression for the sample size n_i for the ith stratum under optimum allocation in stratified random sampling assuming a suitable cost function.

34. Explain Neymann's allocation in stratified sampling and derive the variance of the population total.

35. Give an unbiased estimate of the population proportion and its unbiased estimated variance when random sampling is with stratification.

36. In a linear systematic sample for N = nk, show that the sample mean is an unbiased estimator of the population mean.

37. Distinguish between Ratio estimate and linear regression estimate.

38. Show that the ratio estimators are biased.

(6×4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. Discuss briefly on the organizational aspects of a sampling survey pointing out the pros and cons.

40. Show that the sample mean $\bar{y}$ is an unbiased and consistent estimate of the population mean in SRS and show that $V_{SRSWOR}(\bar{y}) = \frac{N-n}{N-1} V_{SRSWR}(\bar{y})$.

41. Briefly explain the precision of the estimate of the population mean under any three basic probability sampling methods.

42. For populations with a linear trend, prove with usual notations

$$V(\bar{y}_{st}) : V(\bar{y}_{sy}) : V(\bar{y}_{ran}) = \frac{1}{n} : 1 : n.$$

43. Define linear systematic sampling. Show that the higher heterogeneity makes the estimator more efficient in a linear systematic sample.

44. Compare the efficiency of the regression estimator with those based on mean per unit and ratio estimation procedure.

(2 × 15 = 30 Marks)

(Pages : 3) **N – 1392**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme Under CBCSS

Statistics

Core Course – IX

ST 1641 – DESIGN OF EXPERIMENTS AND VITAL STATISTICS

(2014 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** carries **1** mark.

1. Large number of replications be taken if experimental units are ____________.

2. The ratio of births to deaths in a year is called ____________.

3. Define a stable population

4. If Net reproduction rate(NRR) > 1, the population of a country will very likely ____________.

5. If there is a single missing observation in a randomized block design with 4 blocks and 5 treatments, the error degrees of freedom will be

6. The error degrees of freedom for a Latin square design with 4 rows is _________

7. The number of times a treatment is repeated in an experiment is called its ____________.

8. The most commonly used experimental design is ____________.

P.T.O.

9. Define crude birth rate

10. What is standard population?

(10 $\times$ 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Define estimability of a parametric function.

12. Define relative efficiency of a design with respect to other.

13. What is the role of randomization in design of experiments?

14. Mention the advantages of a Randomised block design over CRD.

15. Explain ANOVA.

16. What is a Randomized block design?

17. How can you calculate the specific death rate for a specific section of population?

18. What is crude death rate?

19. What is total fertility rate?

20. Define GMFR.

21. Define a stationary population.

22. List three uses of life tables.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. **Each** question carries **4** marks.

23. Explain Gauss -Markov set up.

24. Explain briefly the basic principles of experimentation.

25. Discuss the efficiency of Randomized block design over Completely randomized design.

26. Explain the procedure of obtaining the estimate of one missing observation in Latin square design.

27. Explain the procedure to collect vital statistics through registration system.

28. Explain how specific death rates are better than crude death rates.

29. Describe force of mortality. What is graduation of mortality rates.

30. Explain how the principle of local control is adopted in Latin Square design.

31. What improvement is brought out by net reproduction rate over gross reproduction rate?

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. **Each** question carries **15** marks.

32. Describe the analysis of a randomized block design.

33. State and prove Gauss Markov theorem.

34. What are the various components of a complete life table. Also mention the various uses of life tables.

35. What is Latin square design? Write down the model for its analysis and also the ANOVA table. Obtain the relative efficiency of LSD compared to RBD treating columns as blocks.

(2 $\times$ 15 = 30 Marks)

(Pages : 3) **N – 1393**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Statistics

Core Course – IX

ST 1641 : DESIGN OF EXPERIMENTS AND VITAL STATISTICS

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Write the mathematical model for a two way Analysis of variance.

2. What are the assumptions of errors in experimental models?

3. Define a randomized design.

4. Which design do you prefer if the experimental units are homogeneous?

5. Define a LSD (Latin Square Design).

6. What effects are measured in factorial experiments?

7. Define Demography.

8. What is a cohort?

9. Define force of mortality.

10. What is crude Birth Rate?

(10 $\times$ 1 = 10 Marks)

P.T.O.

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. What is a Randomized Block Design(RBD)?
12. Give the statistical model(model only) for a Completely Randomized Design CRD with one observation per cell.
13. What do you mean by local control?
14. Explain the advantages of LSD over RBD.
15. What is the importance of a Latin Square Design?
16. How can you calculate the sum of squares for analysis of variance of a LSD?
17. Write the expression for the efficiency of a Randomized Block Design over CRD.
18. What are factorial experiments?
19. What are the effects measured in factorial experiments?
20. What is the function of Sample Registration System of India?
21. Distinguish between curate (curtailed) expectation and complete expectation.
22. What are the methods of standardization of data?
23. Name three methods of constructing an abridged life table.
24. Define central mortality rate.
25. Distinguish between symmetrical and asymmetrical factorials.
26. Give the important measures of fertility.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. Discuss the technique of Analysis of variance for one-way classification.
28. Explain the basic principles of experimentation.

29. Outline the analysis of a data with a single missing value of a $k \times k$ Latin square design.

30. What do you mean by mutually orthogonal Latin squares?

31. What is confounding?

32. Write the set of orthogonal contrasts for main effects and interactions in a 2^3 factorial experiment.

33. Specific Death Rate is better than Crude Death Rate. Justify.

34. Distinguish between stable and stationary population.

35. What is the significance of IMR in population studies?

36. Establish the relation between the life table functions q_x, the probability of dying within one year after attaining age x and m_x, the probability of dying a person whose exact age is not known but lies between x to $(x + 1)$ years(central mortality rate).

37. Distinguish between NRR and GRR.

38. In what way construction of a complete life table differ from that of an abridged life table?

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. Characterize a Completely Randomized Design. What are the merits of CRD?

40. Describe the analysis of a LSD and sketch the ANOVA table.

41. Explain the Yates' method of analysis for a 2^2 factorial experiment.

42. Discuss the various uses of vital statistics for a country.

43. Given the age returns for the two ages x = 9 years and $x+1=10$ years with the life table values as l_9 = 75824, l_{10} = 75362, d_{10} = 418, T_{10} = 4953195. Give the complete life table for the two ages 9 and 10 of the persons.

44. Explain the GFR and the information gathered by it. How the information is improved by Age Specific Fertility Rate and by Total Fertility Rate?

(2 × 15 = 30 Marks)

(Pages : 3)

N – 1394

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Statistics

Core Course – X

ST 1642 : APPLIED STATISTICS

(2014 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

(Use of calculators are allowed)

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Which type of bias is present in Paasche's index number?
2. What is Price relative?
3. Index number for the base year will be ——————.
4. Write any one formula for simple Index number.
5. Write any one use of time series.
6. Define time series with an example.
7. Give examples for random factor in a time series.
8. When was the last census in India held?
9. What does NSSO stand for?
10. Define Census.

(10 $\times$ 1 = 10 Marks)

P.T.O.

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Define Index number.
12. What is the difference between Weighted and Unweighted Index numbers?
13. Write any two uses of Index numbers.
14. Define Consumer Price Index number.
15. What do you mean by time series?
16. What are the components of time series?
17. Define Additive model in time series.
18. What do you mean by Secular Trend?
19. Write any two demerits of moving average method.
20. What are the methods of eliminating seasonal variation?
21. Explain CSO.
22. What is meant by National income?

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

23. Explain the tests to be satisfied by an ideal Index number.
24. What is the difference between seasonal and cyclical variations. Give examples.
25. Define Fisher Index number. Show that it is ideal.
26. Estimate trend from the following data :

Year :	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
U_t :	77	88	94	85	91	98	90	88	80	73

27. Explain moving average method.

28. Explain chain base index number. What are its advantages over fixed base method?

29. Distinguish between base shifting and splicing.

30. Explain Defacto and Dejure methods.

31. What are different Government agencies undertaking census operations?

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

32. Fit a straight line trend to the data

Year :	1992	1994	1996	1998	2000	2002	2004
Production :	77	81	88	94	94	96	98

Estimate the production in 2000.

33. Find Laspeyer's Paasche's and Fishers Index numbers from the following data :

Items	Price	Quantity	Price	Quantity
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24
E	8	40	12	36

Show that Fisher's Index is an ideal index number.

34. Describe the functions and scope of Indian official statistics.

35. Distinguish between Price and Quantity index numbers. Explain the construction of general price index numbers.

(2 $\times$ 15 = 30 Marks)

(Pages : 6) **N – 1395**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Statistics

Core Course – X

ST 1642 – APPLIED STATISTICS

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Index numbers is a :

   (a) measure of relative changes

   (b) a special type of an average

   (c) a percentage relative

   (d) all the above

2. The price index as the arithmetic mean of Laspeyre's and Paasche's indices was expounded by

   (a) Kelly (b) Irving Fisher

   (c) Drobish and Bowley (d) Walsh

P.T.O.

3. If the index number is independent of the units measurements, then it satisfies :

   (a) time reversal test

   (b) factor reversal test

   (c) unit test

   (d) all the above

4. Trend in a time series means

   (a) long-term regular movement

   (b) short -term regular movement

   (c) both (a) and (b)

   (d) neither (a) nor (b)

5. The component of a time series which is attached to short-term fluctuations is :

   (a) seasonal variation

   (b) cyclic variation

   (c) irregular variation

   (d) all the above

6. What is NSSO?

   (a) National Social Science Office

   (b) National Social Study Office

   (c) National Security Science Office

   (d) National Sample Survey Office

7. For consumer price index, the price data should be collected from ————.

8. The CSO is headed by ————.

9. Given the trend equation $Y = 118.5 + 2.2X + 1.4\ X^2$ with origin 2000 the trend equation with origin 2001 is ————.

10. Quarterly fluctuations observed in time series ———— represent variation.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. What is NSO? What are its different wings?

12. Explain De-facto method.

13. What is a price relative?

14. What is splicing?

15. Give any two limitations of index numbers.

16. Explain briefly the concept of cost of living index number.

17. Explain the precautions that we have to take to fix ‘base year’ to calculate index number.

18. Explain the components of time series.

19. Illustrate the linear and non linear trend in time series.

20. Explain the mathematical models in time series.

21. What are the normal equations to fit a straight tine $y = a + bx$.

22. What are the merits and demerits of semi average method?

23. How can we obtain the statistics of crop yields?

24. What is circular test?

25. Explain briefly the concept of whole sale price index number.

26. Define moving average.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. What are the main functions of NSSO?

28. What are the types of census enumeration?

29. Elucidate the uses and limitations of time series analysis.

30. What do you mean by 'Business cycle'? Explain.

31. What is method of least squares? What are normal equations of a straight line?

32. What is a time series? Explain with examples.

33. Give two examples each to

    (a) Seasonal variation

    (b) Irregular variation.

34. Explain briefly how the index numbers are used to measure the purchasing power of money?

35. Differentiate Laspeyre's from Paasche's index number. Among these which one is superior and why?

36. Distinguish between simple index number and weighted index number. Mention any two applications of weighted index number.

37. Explain and illustrate :

    (a) Base shifting

    (b) Deflating

38. Why index numbers are called economic barometers? Explain.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. Describe the steps involved in Ratio to moving average method of measuring seasonal indices.

40. Explain factor reversal test and time reversal test. Show that Fishers Ideal index number satisfies both these tests.

41. Explain the role of index numbers in the socio-economical analysis. What are the main factors to be cared while constructing an index number?

42. What you mean by Statistics of Labour and Employment. What are the methods used for national income estimation?

43. Below are given the figures of production (in thousand quintals) of a sugar factory:

Year	2001	2002	2003	2004	2005	2006	2007
Production	80	90	92	83	94	99	92

    (a) Fit a straight line trend to these figures.

    (b) Plot these figures on a graph and show the trend line.

44. From the following data of wholesale prices of wheat for ten years construct index number taking

(a) 1998 as base and

(b) by chain base method

Year	Price of Wheat	Year	Price of Wheat
1998	50	2003	78
1999	60	2004	82
2000	62	2005	84
2001	65	2006	88
2002	70	2007	90

(2 $\times$ 15 = 30 Marks)

(Pages : 3) **N – 1396**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Statistics

Core Course – XI

ST 1643 : OPERATIONS RESEARCH AND STATISTICAL QUALITY CONTROL

(2014 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

(Answer **all** questions. Each question carries **1** mark)

1. The percent of the sample means will have values that are within $\mu \pm 3\sigma$ is ——————.

2. When do you say a process is under statistical control?

3. State the control limits of *R* chart.

4. Convex set of equations is included in the —————— region for linear programming equations.

5. Define surplus variables.

6. What is meant by *np* chart?

7. The average percentage of defectives remaining in an outgoing lot is known as ——————.

P.T.O.

8. Define double sampling plan.

9. SQC techniques were developed by ————.

10. In standard form of linear programming problems, all constraints are expressed as ————.

(10 $\times$ 1 = 10 Marks)

SECTION – B

(Answer **any eight** questions. Each question carries **2** marks.)

11. What are the assumptions of a LPP?

12. Distinguish between process control and product control.

13. What is degeneracy in transportation problem?

14. What are the two types of control chart for variables?

15. How the assignment problem can be viewed as a linear programming problem?

16. Define :
    (a) Basic Solution
    (b) Feasible solution.

17. Define OC curve.

18. What is meant by LTPD?

19. What do you mean by Average Total Inspection?

20. Explain quality of a lot.

21. What is meant by optimal solution?

22. What do you meant by natural tolerance limits?

(8 $\times$ 2 = 16 Marks)

SECTION – C

(Answer **any six** questions. Each question carries **4** marks.)

23. State the general linear programming problem. Define basic solution and optimal solution in LPP.

24. Explain modified control limits.

25. Briefly explain the graphical procedure to solve a LPP.

26. State duality in LPP. Explain its advantages.

27. The average number of defectives in 22 sampled lots of 2000 rubber belts each was found to be 16%. Indicate how to construct the relevant control chart.

28. Write down the merits and demerits of double sampling plan.

29. Explain any one method of finding initial BFS for transportation problem.

30. State any four advantages of control charts.

31. Differentiate between charts for variables and charts for attribute.

(6 $\times$ 4 = 24 Marks)

SECTION – D

(Answer **any two** questions. Each question carries **15** marks.)

32. Write short note on Simplex method of solving LPP. Explain how do you recognize optimality in Simplex method.

33. (a) Distinguish between producer's risk and Consumer's risk.

    (b) Explain ASN, OC and AOQ for single sampling plan.

34. What is the unbalanced Assignment problem? How is it solved by the Hungarian method?

35. What are control charts for variables and attributes? Also explain how you will construct the control charts for variables and attributes.

(2 $\times$ 15 = 30 Marks)

(Pages : 6) **N – 1397**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Statistics

Core Course

ST 1643 : OPERATION RESEARCH AND STATISTICAL QUALITY CONTROL

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

Instructions : Statistical tables and calculator are allowed.

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define unbounded solution.
2. Define optimum basic feasible solution.
3. When do we say that a basic feasible solution is non-degenerate?
4. What is the use of least cost method?
5. Write any method for statistical process control.
6. Write the modern definition of quality.
7. Give an example for chance cause of variation.

8. When do we use p chart?

9. Write the distribution based on which the statistical principle of c chart are underlying.

10. Write the average sample number of single sampling plan when the sample size is 100.

(10×1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. List the assumptions of a linear programming problem.

12. Explain the significance of artificial variable.

13. Discuss degeneracy in TPP?

14. Distinguish between slack variables and surplus variables.

15. What is meant by transportation problem?

16. Define feasible solution.

17. Write the dual of the LPP:

Maximise $Z = x_1 - x_2 + 3x_3$

Subject to

$$x_1 + x_2 + x_3 \leq 10$$
$$2x_1 - x_2 - x_3 \leq 2$$
$$2x_1 - 2x_2 - 3x_3 \leq 6$$
and $x_1, x_2, x_3 \geq 0$

18. Describe an OC curve. Write any one use of OC curve.

19. What is meant by natural tolerance limits?

20. Define statistical quality control.

21. Write the control limits of range chart when the parameter values are known.

22. Describe any method to construct rational subgroups.

23. Define assignable cause of variation.

24. Define acceptance sampling.

25. Define AOQ and AOQL.

26. Distinguish between AQL and LTPD.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. Solve the following LPP using graphical method.

Minimize $Z = -x_1 + 2x_2$

Subject to

$-x_1 + 3x_2 \leq 10$
$x_1 + x_2 \leq 6$
$x_1 - x_2 \leq 2$ and
$x_1, x_2 \geq 0$

28. Discuss the mathematical formulation of a linear programming model.

29. Prove that the dual of the dual is primal.

30. Describe two phase method for solving an LPP.

31. Write the steps for solving an LPP using Big M method.

32. Compare consumer's risk and producers' risk. How do they influence the selection of control limits?

33. Discuss the statistical principle of a control chart.

34. Discuss the applications of statistical quality control techniques in industry.

35. 12 samples of 200 bulbs each were examined and the number of defective bulbs in each sample are given. Set up a control chart for fraction nonconformities using these data.

Sample Number	1	2	3	4	5	6	7	8	9	10	11	12
Number of defectives	3	2	4	3	2	0	3	1	1	2	4	0

36. Discuss the construction and applications of *d* chart.

37. Derive the OC function of single sampling plan. Discuss the effect of sample size and acceptance number on the OC curve of SSP.

38. Discuss the multiple sampling plan.

($6 \times 4 = 24$ Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. Solve by simplex method

Max $Z = 3x_1 + 5x_2 + 4x_3$

Subject to

$2x_1 + 3x_2 \le 8$
$2x_2 + 5x_3 \le 10$
$3x_1 + 2x_2 + 4x_3 \le 15$ and
$x_1, x_2, x_3 \ge 0$

40. Describe assignment problem. Explain the Hungarian method for solving the assignment problem.

41. Explain North West corner method and Vogel's approximation method for finding the initial feasible solution of a transportation problem.

42. Construct control chart of mean and range for the following data and comment on the state of control.

Sub group	1	2	3	4	5	6	7	8	9	10	11	12	13
x_1	459	443	457	469	443	444	445	446	444	432	445	456	459
x_2	449	440	444	463	457	456	449	455	452	463	452	457	445
x_3	435	442	449	453	445	456	450	449	457	463	453	436	441
x_4	450	442	444	438	454	457	445	452	440	443	438	457	447

$(A_2 = 0.729,\ D_3 = 0,\ D_4 = 2.282)$

43. (a) Explain the construction of *c* chart and *u* chart.

(b) Following table presents the number of nonconformities observed in 20 successive samples of 100 printed circuit boards. Draw a *c* chart for the data and comment on the state of control.

Sample Number	1	2	3	4	5	6	7	8	9	10
Number of non conformities	20	24	16	6	15	11	27	20	31	24
Sample Number	11	12	13	14	15	16	17	18	19	20
Number of non conformities	21	10	18	13	22	19	39	16	24	30

44. Compare single sampling and double sampling plans. Compute the probability of acceptance of a double sampling plan with acceptance numbers $c_1 = 1$, $c_2 = 3$ and sample sizes $n_1 = 50$, $n_2 = 100$ when the lot fraction defective is 0.05.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 1400**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Statistics

Elective Course

ST 1661.2 : STOCHASTIC PROCESSES

(2014 and 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Find the probability generating function (pgf) of Geometric distribution.

2. What is state space?

3. Define Markov process.

4. Define transition probability matrix.

5. Define auto covariance.

6. What is spectral density?

7. State Chapman-Kolmogorov equation.

8. Define Gaussian process.

P.T.O.

9. What do you mean by a transient state?

10. Give an example of Poisson process.

(10×1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Define pgf. Explain the method to find mean from it.

12. When a stochastic process is said to have independent increments?

13. Define ergodic state of a Markov chain.

14. Define period of a state in a Markov chain. Give an example of it.

15. Define moving average process.

16. When will you say that a stochastic process $\{x_n; n = 0, 1, 2, ...\}$ is a Markov chain.

17. Define a branching process. Give an example of it.

18. Define first order auto regressive process.

19. What is a compound poisson process?

20. What is meant by a trend in time series?

21. State ergodic theorem.

22. Let $\{X_n; n = 0, 1, 2, ...\}$ be a branching process and the corresponding offspring distribution has the pgf $P(s) = \frac{2}{3} + \frac{s + s^2}{6}$. Find the probability of extinction of $\{X_n\}$.

(8×2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

23. Define covariance stationary process. Illustrate with an example.

24. What is meant by n-step transition probabilities of a Markov chain? Calculate $P_{12}^{(2)}$ if $P = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.4 & 0.4 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}$ where the state space is $\{1, 2, 3\}$.

25. Define probability of extinction. Describe the method to find this probability.

26. If $X_1(t)$ and $X_2(t)$ are two independent poisson process with intensity parameters μ_1 and μ_2 respectively, find the conditional distribution of $X_1(t)$ given $X_1(t) + X_2(t)$.

27. Define Poisson process. How it is related to Uniform distribution.

28. Let $\{X_n; n \geq 0\}$ be a Markov chain with states 0, 1 and 2. The transition probability matrix is $\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ with initial distribution $P(X_0 = i; i = 0, 1, 2) = 1/3$. Find $P\{X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2\}$.

29. Define components of time series with examples.

30. Explain the classification of stochastic processes with respect to state space and time with suitable examples.

31. If $P(s)$ is the off spring distribution associated with a branching process $\{X_n\}$ and $P_n(s)$ is that of X_n, Show that $P_n(S) = P(P_{n-1}(s)) = P_{n-1}(P(s))$.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

32. Let $X_i, i = 1, 2, ...$ be identically and independently distributed random variables with $P\{X_i = k\} = p_k$ and pgf $P(s)$ and let $S_N = X_1 + X_2 + ... + X_N$ and N is a random variable independent of $X_i' s$. Find the distributions of S_N in terms of pgf.

33. (a) Explain branching process. Show that it is stationary.

    (b) The offspring distribution of a branching process is given by $P_0 = \frac{1}{3}, P_1 = \frac{2}{6}, P_2 = \frac{1}{3}$, Find the probability of extinction.

34. Explain the postulates of a Poisson process. Show that for a Poisson process $\{N(t)\}$, $P(N(t) = n)$ is the Poisson pdf of distribution.

35. Let $\{X_n, n \geq 0\}$ be a Markov chain with state space $\{0, 1, 2, 3\}$ having tpm $\begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$. Classify the Markov chain and its states.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 1401**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Statistics

Elective Course

ST 1661.2 : STOCHASTIC PROCESSES

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Define the state space of a Stochastic Process.
2. When do you say that a Stochastic Process is a continuous time process?
3. Find the probability generating function of a Binomial Random variable.
4. When do you say two states of a Markov Chain are communicative?
5. Define a Markov chain.
6. When is a transition probability matrix (TPM) said to be stochastic?
7. Define a compound Poisson process.
8. Define a Branching process.
9. What is stationarity in Stochastic process?
10. What is irregular variation in a time series data?

(10 $\times$ 1 = 10 Marks)

P.T.O.

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. If X and Y are independent Poisson random variables with parameters λ and μ respectively, then what is the distribution of $X + Y$?

12. Establish the expression to get the variance from a probability generating function.

13. If $f(x, y) = Ae^{-(x+y)}, 0 < x, y < \infty$, is the joint probability density function of x and y, find A and also find the marginal pdfs of X and Y and check their independence.

14. Show that recurrence is a class property.

15. When do you say a Markov Chain is irreducible?.

16. Define absorbing Markov Chain with an example.

17. What is the period of a particular state in a Markov Chain?

18. What are the properties of a TPM?

19. For an irreducible Markov Chain, if the stationary distribution exists, then it is unique. Justify.

20. State the ergodic theorem.

21. Distinguish between strict sense and weak sense stationarity.

22. What are the usual Mathematical models used in time series analysis?

23. Define exponential smoothing in a time series data.

24. What is the significance of autocorrelation in time series analysis?.

25. Define the first order autoregressive model.

26. Define the probability of extinction in a branching process.

(8 $\times$ 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. Give an example of a discrete state branching process.

28. Obtain the conditional densities from the joint pdf $f(x, y) = 3 - x - y$, $0 < x, y \leq 1$. Also check the independence of X and Y.

29. Find the Probability generating function (PGF) of a Geometric random variable.

30. For an integer valued r.v X, with $P(X = n) = p_n$ and $P(X \leq n) = q_n$ so that $\sum_{i=0}^{n} p_i = q_n$, then prove that $\sum_{n=0}^{\infty} P(X \leq n)s^n = \frac{G_X(S)}{1-s}, |s| \leq 1$, where $G_X(s)$ is the PGF of X.

31. Explain the various classifications of Stochastic Processes.

32. Distinguish between recurrent and transient states of a Markov Chain.

33. A Markov Chain $\{X(t), t = 0, 1, 2, ...\}$ defined on the state space {1,2,3} has the following TPM. Find the stationary distribution of the chain.

$$P = \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$

34. Show that for an irreducible Markov Chain, the stationary distribution, if exists, is unique.

35. Discuss on the components of a time series data.

36. Give the names of different methods of measuring trend in time series analysis.

37. Show that for a Gaussian Stochastic process both weak and strong stationarity are equivalent.

38. Establish the Branching process recursion formula based on the probability generating function.

(6 $\times$ 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. Let $X_1, X_2, ...$ be a sequence of independent and identically distributed random variables with common PGF as $G_X(S)$. Let N be a random variable independent of the random variables X_i's with PGF as $G_N(s)$ and let $T_N = \sum_{i=1}^{N} X_i$. Then show that the PGF of T_N is $G_{T_N}(s) = G_N(G_x(s))$. Also compute the mean of T_N.

40. A Markov Chain defined with state space S = {1,2,3,4,5} has the following transition probability matrix P. Find (a) all closed classes, (b) irreducible classes, (c) recurrent and (d) transient states.

$$P = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccccc} \;\;1 & \;\;\;2 & \;\;\;3 & \;\;\;4 & \;\;\;5 \end{array} \\ \begin{pmatrix} 0.3 & 0.4 & 0.4 & 0 & 0 \\ 0 & 0 & 0.3 & 0.4 & 0.4 \\ 0 & 0.3 & 0 & 0.3 & 0.4 \\ 0 & 0 & 0 & 0.4 & 0.6 \\ 0 & 0 & 0 & 0.6 & 0.4 \end{pmatrix} \end{array}$$

41. How do you fit a trend line by the method of least squares in a time series analysis? Also mention the merits and demerits of the method.

42. Define Poisson process. State the important postulates of the Poisson process. If the arrival process is Poisson, then what is the distribution of the inter arrival (waiting) times?

43. Let $\{Z_0 = 1, Z_1, Z_2, ...\}$ be a Branching process with family size Y having a Binomial(2, 1/4) distribution. Find the probability that the process will eventually die out.

44. Explain Galton-Watson branching process. Let μ be the expected number of offsprings in each generation in a Galton-Watson branching process. Show that, if $\mu \leq 1$, the process dies out with probability one.

(2 $\times$ 15 = 30 Marks)

(Pages : 4) **N – 1402**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Statistics

Elective Course

ST 1661.3 : INVENTORY CONTROL AND QUEUING THEORY

(2014 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each question carries **1** mark.

1. What is single item inventory system?

2. Single period EOQ model for uncertain demand is usually known as ————.

3. What is backorder stockout case?

4. If the demand rate is 100 units per day and the optimum cycle length is 2.5 time units, what is the optimum order quantity?

5. Customers form a queue are selected for service according to certain rule is known as ————.

6. In a single server Poisson queuing system with traffic intensity 0.4, what is the probability that a person has to wait for service?

7. The behavior of customers after joining the queue, wait for some time but leave before being served is known as ————.

P.T.O.

8. Give two examples of slow moving inventory items.

9. Inventory kept on hand against stockout due to unforeseen events is known as ____________.

10. What is ideal time in queuing theory?

(10 $\times$ 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. What is classic EOQ model?

12. Point out the steps of inventory model building.

13. What is an inventory policy? Point out its objective.

14. Define
    (a) lead time and
    (b) reorder level.

15. Distinguish between continuous review and periodic review inventory systems.

16. Give any two applications of queuing theory.

17. Distinguish between transient and steady state distributions.

18. What are commonly used measures of performance in a queuing system?

19. Arrivals at telephone booth are considered to be Poisson with an average time of 8 minutes between one arrival and the next. The length of phone call is assumed to be exponential with mean 2 units. What is the probability that queue size exceeds 5?

20. Distinguish between parallel and series service channels in queuing system.

21. Define M/G/1 queuing system.

22. What is Markov process? Define Markov chain.

(8 $\times$ 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries **4** marks.

23. Explain newsboy problem.

24. Describe briefly the importance of demand and lead time in inventory management problem.

25. The demand for a particular item is 18000 units per year. The holding cost per unit is Rs.1.20/- per year and the procurement cost is Rs.400/- per unit. Shortages are not allowed and instantaneous replacement rate. Determine optimum order quantity and optimum time between orders.

26. Define inventory system. Distinguish between stochastic and deterministic inventory systems.

27. Explain the role of probability distributions in queuing theory.

28. Define :
    (a) M/M/1 queuing system with finite capacity and
    (b) Embedded Markov chain.

29. Obtain expected number of customers in single server Poisson queuing system with infinite capacity. What is the average queue length?

30. Explain $M/E_k/1$ queuing system and its characteristics.

31. Consider a single server Poisson queuing system with mean arrival rate 3 calling units per unit time and expected service time is 0.25 hour, The maximum permissible unit in the system is 2. Obtain the steady state probability distribution of calling units in the system and its expected value.

(6×4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries **15** marks.

32. (a) Explain in detail the various costs and revenues associated with an inventory management system.

    (b) The annual demand of a product is 10000 units. Each unit costs Rs.100 if the orders placed in below 200 units of quantities but for orders of 200 and above, the price is Rs.95. The annual inventory holding costs is 10% of the value of the item and the ordering cost is Rs.5 per order. Find the economic lot size.

33. Stating assumptions,

    (a) develop a single period model with uncertain demand and no setup cost;

    (b) A baking company sells cake by kilogram weight. It makes a profit of Rs.5 on every kilogram sold on the day it is baked. It disposes of all cakes not sold on the date it is baked, to a loss of Rs.1.20 per kg. If demand is known to uniform between 2000 and 3000 kilograms, determine the optimum daily amount baked.

34. Derive the difference equations and obtain the steady state solution for the queuing model (M/M/c) with infinite capacity. Also find expected number of customers waiting in the queue.

35. (a) Describe the basic characteristics and structure of queuing system.

    (b) Obtain the probability distribution of waiting time under steady state for the (M/M/1): (∞/FIFO) model.

($2 \times 15 = 30$ Marks)

(Pages : 4) **N – 1403**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme Under CBCSS

Statistics

Elective Course

ST 1661.3 – INVENTORY CONTROL AND QUEUING THEORY

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. What do you mean by inventory?
2. Define set up cost.
3. What is price break?
4. What do you mean by a stochastic inventory model?
5. Define safety or Buffer stock?
6. Define a queuing model.
7. What do you mean by service time?
8. Define queue length.
9. What is bulk arrivals?
10. Briefly describe the $M \mid G \mid 1$ queuing system.

(10 $\times$ 1 = 10 Marks)

P.T.O.

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Define EOQ.
12. Name the uncontrolled factors in inventory control.
13. What is inventory turnover?
14. What are the various costs associated with inventory control?
15. What are deterministic inventory models?
16. Define lead time in inventory control. How it affects the stock?
17. Illustrate the Kendall notation of a queuing model.
18. Distinguish between waiting time in the queue and waiting time in the system.
19. If the arrival and departure rates in a M/M/1 queue are 1/2 per minute and 2/3 per minute respectively, find the average waiting time of a customer in the queue.
20. Mention some applications of queuing theory.
21. What is the steady state and transient state probability of a queue system?
22. Describe traffic intensity.
23. Explain M/M/1 model with unlimited channel capacity.
24. Give the postulates of a Poisson process.
25. How do you explain a queuing model as a birth-death process?
26. Define multiple server queuing system.

($8 \times 2 = 16$ Marks)

SECTION – C

Answer **any six** questions. **Each** question carries **4** marks.

27. What are important deterministic inventory models?
28. Briefly explain the fundamental deterministic inventory model with instantaneous stock replenishments and without shortages.

29. Describe the Newspaper boy problem.

30. Find the EOQ of inventory problem with shortages and finite instantaneous production.

31. Discuss on the probabilistic inventory control models.

32. What are the operating characteristics of a queuing system

33. Name four important queue disciplines.

34. Show that when the arrival process is Poisson, the inter arrival time follows exponential distribution.

35. Give the Little's formulae.

36. Show that for a single server Poisson arrival process, the probability that exactly n units in the queuing system is $P_n = \rho^n(1-\rho)$, where ρ is the traffic intensity.

37. A TV repairman finds that the time spend on his job has an exponential distribution with mean 30 minutes. If the repair sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hours per day, what is the repairman's expected idle time each day? How many jobs are expected ahead of the set just brought?

38. Discuss the (M/M/1) queue model in the steady state.

(6×4 = 24 Marks)

SECTION – D

Answer **any two** questions. **Each** question carries **15** marks.

39. A textile mill buys its raw materials from a vendor. The annual demand of the raw materials is 9000 units. The ordering cost is Rs. 100 per order and the carrying cost is 20% of the purchase price per unit per month, where the purchase price per unit is Rs.1. Then find

    (i) The EOQ,

    (ii) Total cost,

    (iii) Number of orders per year and

    (iv) Time between two consecutive orders.

40. Formulate and solve the purchase inventory problem with constant demand, instantaneous replenishments without shortages and having a single price break.

41. Explain the stochastic inventory model with uncertain uniform discrete demand and zero lead time.

42. The arrival rate of customers at a banking counter follows Poisson distribution with a mean of 45 per hour. The service rate of the counter clerk also follows Poisson distribution with a mean of 60 per hour.

    (i) What is the probability of having Zero customer in the system?,

    (ii) What is the probability, of having 5 customers in the system?

    (iii) Find the average number of customers in the queue,

    (iv) Find the average number of customers waiting in the system,

    (v) Find the average waiting time per customer in the queue and

    (vi) Find the average waiting time per customer in the system.

43. Obtain the steady-state difference equations of the birth and death model. How do you solve the this model to obtain the probability distribution of the queue length?

44. Describe the various cost models in queuing.

(2 $\times$ 15 = 30 Marks)
