(Pages : 3) **P – 5243**

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, September 2022

Mathematics

MM 221 – ABSTRACT ALGEBRA

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 75

SECTION – A

Answer **any five** questions. **Each** question carries **3** marks.

1. Let $G = U(16)$, $H = \{1, 15\}$ and $K = \{1, 9\}$. Are H and K isomorphic? Are G/H and G/K isomorphic?

2. Prove that a group of order 105 contains a subgroup of order 35.

3. Express $x^8 - x$ as a product of irreducible polynomials over $\mathbb{Z}_2$.

4. Construct a field of order 9.

5. Find $\Phi_{12}(x)$.

6. If a and b are constructible numbers, give a geometric proof that $a + b$ is constructible.

7. Show, by an example, that if the order of a finite abelian group is divisible by 4, the group need not have a cyclic subgroup of order 4.

8. Find the minimal polynomial for $1 + \sqrt[3]{2} + \sqrt[3]{4}$ over $\mathbb{Q}$.

(5 × 3 = 15 Marks)

P.T.O.

SECTION – B

Answer **all** questions. **Each** question carries **12** marks.

9. (A) (a) Let G be an abelian group of prime-power order and let a be an element of maximal order in G. Prove that G is the internal direct product of $\langle a \rangle \times K$ for some subgroup K in G. **9**

(b) Show, by example, that in a factor group G/H it can happen that $aH = bH$ but $|a| \neq |b|$. **3**

OR

(B) (a) Prove that the order of an element of a direct product of a finite number of finite groups is the least multiple of the order of the components of the element. **7**

(b) Express $U(165)$ as an internal direct product of proper subgroups in two different ways. **5**

10. (A) (a) Prove that any two Sylow p-subgroups of a finite group G are conjugate. **7**

(b) Prove that a group of order 175 is abelian. **5**

OR

(B) (a) Suppose that G is a group of order 60 and G has a normal subgroup N of order 2. Prove that G has a cyclic subgroup of order 30. **6**

(b) Prove that if G is a finite group and H is a proper normal subgroup of largest order, then G/H is simple. **6**

11. (A) (a) Prove that a finite extension of a finite extension is finite. **8**

(b) Find the splitting field for $x^3 + x + 1$ over $\mathbb{Z}_2$. **4**

OR

(B) (a) Let $f(x)$ be an irreducible polynomial over a field F and let E be a splitting field of $f(x)$ over F. Prove that all the zeros of $f(x)$ in E have the same multiplicity. **7**

(b) Find the degree and a basis of the splitting field of $x^6 + x^3 + 1$ over $\mathbb{Q}$. **5**

12. (A) (a) Prove that the maximum degree of any irreducible factor of $x^8 - x$ over $\mathbb{Z}_2$ is 3. **6**

(b) Prove that, for each positive divisor m of n, $GF(p^n)$ has a unique subfield of order p^m. Find the number of subfields of $GF(625)$. **6**

OR

(B) (a) Prove that an angle θ is constructible if and only if $\cos\theta$ is constructible. **8**

(b) Prove that a 40° angle is not constructible. **4**

13. (A) (a) Find the Galois group of $\mathbb{Q}(\sqrt[4]{2}, i)$ over $\mathbb{Q}$. **6**

(b) Prove that $\Phi_{2n}(x) = \Phi_n(-x)$ for all odd positive n. **6**

OR

(B) (a) Let N be a normal subgroup of a group G. If both N and G/N are solvable, prove that G is solvable. **6**

(b) Prove that $\Phi_n(x) \in \mathbb{Z}[x]$. **6**

(5 × 12 = 60 Marks)

(Pages : 3) **P – 5244**

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, September 2022

Mathematics

MM 222 — REAL ANALYSIS II

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 75

PART – A

Answer **any five** questions. Each question carries **3** marks.

1. Define Lebesgue outer measure and prove that it is countably subadditive and translation invariant.

2. Let $f = g$ a.e. where f is a continuous function. Show that ess sup f =ess sup g = sup f.

3. Show that $\int_0^1 \sin x \log x dx = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)(2n)!}$.

4. Show that the derivatives of a continuous function are measurable.

5. Prove that the limit of a pointwise convergent sequence of measurable functions is measurable.

6. Show that if $0 < a < \infty$ and $0 < p < \infty$ then $\log x^{-1} \in L^p(0, a)$.

7. State and prove Jensen's inequality.

8. Show that if $f_n \to f$ in measure and α is any real number, then $\alpha f_n \to \alpha f$ in measure.

(5 × 3 = 15 Marks)

P.T.O.

PART – B

Answer **any** questions choosing either (a) or (b). Each question carries **12** marks.

9. (A) (a) Prove that the interval (a, ∞) is measurable. **3**

(b) Prove that the Lebesgue outer measure of an interval is its length. **9**

OR

(B) (a) Let $< E_i >$ be a sequence of measurable sets. Prove that $m\left(\bigcup E_i\right) \leq \sum mE_i$. If the sets E_i are pairwise disjoint, then prove that $m\left(\bigcup E_i\right) = \sum mE_i$. **6**

(b) Give an example of a measurable set that is not a Borel set. **6**

10. (A) Prove that if *f* is Riemann integrable and bounded over the finite interval $[a, b]$, then f is integrable and $R\int_a^b f\, dx = \int_a^b f\, dx$. What can you say of the converse? Justify. **12**

OR

(B) (a) Prove that if $f \in L(a, b)$ then $F(x) = \int_a^x f(t)\, dt$ is a continuous function on $[a, b]$ and is of bounded variation on $[a, b]$. **6**

(b) If *f* is a finite-valued monotone increasing function defined on the finite interval $[a, b]$, then prove that f' is measurable and $\int_a^b f' dx \leq f(b) - f(a)$.

6

11. (A) Prove that if μ is a σ-finite measure on a ring $\mathbb{R}$, then it has a unique extension to the σ-ring $S(\mathbb{R})$. **12**

OR

(B) If μ is a measure on a σ-ring S, then prove that the class $\overline{S}$ of sets of the form $E \Delta N$ for any sets E, N such that $E \in S$ while N is contained in some set in S of zero measure, is a σ-ring and the set function $\overline{\mu}$ defined by $\overline{\mu}(E \Delta N) = \mu(E)$ is a complete measure on $\overline{S}$ **12**

12. (A) (a) Prove that every function that is convex on an open interval is continuous. **6**

(b) State and prove Minkowski's inequality. Also discuss when equality occurs. **6**

OR

(B) Prove that for $p \geq 1$, $L^p(\mu)$ is a complete metric space. **12**

13. (A) Prove that the signed measure on [[X, S]] has a Jordan decomposition. Show also that this decomposition is unique and minimal. **12**

OR

(B) State and prove the Radon-Nikodym theorem. **12**

(5 $\times$ 12 = 60 Marks)

(Pages : 3) **P – 5245**

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, September 2022

Mathematics

MM 223 – TOPOLOGY II

(2020 Admission onwards)

Time : 3 Hours Max. Marks : 75

PART – A

Answer **any five** questions. **Each** question carries **3** marks.

1. Prove or disprove : countable product of second countable spaces is second countable.

2. Prove that the projection maps $p_i : X \to X_i$, where $X = X_1 \times X_2 \times .... \times X_n$ are continuous.

3. Show that $R/\sim$ is topologically equivalent to a circle.

4. Define T_i-spaces for i = 1, 2 and give an example for a T_1-space which is not T_2.

5. Prove or disprove : product of any family of regular spaces need not be regular.

6. If $f : X \to Y$ then show that f is continuous at $x_0 \in X$ if and only if whenever $\mathscr{F} \to x_0$ in X then $f(\mathscr{F}) \to f(x_0)$ in Y.

7. Prove or disprove : every contractible space is simply connected.

8. Is the set of end points $E = \{a, b\}$ a retract of a closed interval $[a, b]$ where $a < b$? Justify your answer.

(5 × 3 = 15 Marks)

PART – B

Answer **all** questions. Each question carries **12** marks.

9. A. (a) Prove that product of an arbitrary Collection of connected spaces is connected. **6**

(b) Define (i) Weak topology (ii) Projection map (iii) Quotient space. **6**

OR

B. (a) Prove that product of a finite number of compact spaces is compact. **6**

(b) Let X and Y be spaces and $f : X \to Y$ be a continuous function from X onto Y. Prove that the natural correspondence h : $X / \sim \to Y$ defined by $h([x]) = f(x),\ x \in X$ is a homeomorphism if and only if Y has the quotient topology determined by f. **6**

10. A. State and prove Tietze extension theorem. **12**

OR

B. (a) Show that every metric space is normal. **6**

(b) Prove that Sorgenfrey plane is regular but not normal. **6**

11. A. State and prove Tychonoff theorem; prove at least one significant result used in it. **12**

OR

B. (a) Show that $\mathscr{F}$ has x as a cluster point if and only if there is a filter $\mathscr{G}$ finer than $\mathscr{F}$ which converges to x. **6**

(b) If X is a first countable space and $E \subset X$, then show that $x \in \overline{E}$ if and only if there is a sequence (x_n) contained in E which converges to x. **6**

12. A. (a) Let X be a path connected space and x_0, x_1 points of X. Show that the fundamental groups $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are isomorphic. **6**

(b) State and prove covering homotopy property. **6**

OR

B. (a) Show that the homotopy class [c], where c is the constant loop whose only value is x_0, is the identity element for $\pi_1(X, x_0)$. **6**

(b) Prove that the fundamental group $\pi_1(S^1)$ is isomorphic to the additive group $\mathbb{Z}$ of integers. **6**

13. A. (a) If D is a deformation retract of a space X and x_0 is a point of D, show that $\pi_1(X, x_0)$ and $\pi_1(D, x_0)$ are isomorphic. **6**

(b) State and prove Brouwer fixed point theorem. **6**

OR

B. Show that the n-sphere S^n is simply connected for $n \geq 2$. **12**

(5 × 12 = 60 Marks)

(Pages : 3) **P – 5246**

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, September 2022

Mathematics

MM 224 — PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 75

PART – A

Answer any **five** questions. Each question carries **3** marks.

1. Solve the PDE $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

2. Solve by Lagrange's method $\left(\frac{y^2 z}{x}\right)\frac{\partial z}{\partial x} + xz\frac{\partial z}{\partial y} = y^2$.

3. Classify the given PDE $x^2 u_{xx} - 2xyu_{xy} + y^2 u_{yy} + xu_x + yu_y = 0$.

4. Show that the derivative u_x of a solution $u(x, y)$ to wave equation will also be a solution.

5. Find the eigen values of the Integral Equation $y(s) = \lambda \int_0^1 e^{s+t} y(t)\,dt$.

P.T.O.

6. Find the resolvant kernel for the Integral Equation $y(s) = f(s) + \lambda \int_0^1 e^{s-t} y(t) dt$.

7. Show that extremals of the arc length functionals are straight lines.

8. State Hamilton's principle.

(5 × 3 = 15 Marks)

PART – B

Answer **all** questions. Each question carries **12** marks.

9. (A) (a) Solve the partial differential equation $u_x + u_y = 2$ with the initial condition $u(x,0) = x^2$. **9**

(b) State the generalized Transversality condition. **3**

OR

(B) (a) Find the equation of the surface satisfying the PDE $4yu \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + 2y = 0$ and passing through $y^2 + u^2 = 1$, $x + u = 2$. **6**

(b) Solve the PDE $u_x + 3y^{\frac{2}{3}} u_y = 2$ subject to the initial condition $u(x,1) = 1 + x$. **6**

10. (A) (a) Write the d-Alembert's solution to the wave equation $u_{tt} = c^2 u_{xx}$, $u(x,0) = 0$, $u_t(x,0) = \cos x$. **6**

(b) Reduce $u_{xx} = x^2 u_{yy}$ to canonical form. **6**

OR

(B) (a) Solve the initial value problem $u_x + 2u_y = 0$, $u(0,y) = 4e^{-2x}$ using the method of separation of variables. **6**

(b) Sketch the regions in which the PDE $yu_{xx} - 2u_{xy} + xu_{yy} = 0$ is elliptic, parabolic and hyperbolic. **6**

11. (A) Establish the law of conservation of energy of the wave equation that represents the motion of an infinite string. **12**

OR

(B) Solve the diffusion equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ with the initial condition $u(x, 0) = e^{-x}$ using the method of Green's function. **12**

12. (A) Find the resolvent kernel of the Integral Equation $y(s) = f(s) + \lambda \int_0^1 (s + t) g(t) dt$. **12**

OR

(B) Solve the Integral Equation $y(s) = s + \lambda \int_0^1 \left[st + (st)^{\frac{1}{2}} \right] y(t) dt$. **12**

13. (A) Extremize the functional $\mathcal{J}[y(x)] = \int_0^{\frac{\pi}{2}} \left[(y')^2 - y^2 \right] dx; y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 1$. **12**

OR

(B) Find the minimal surface of the functional $\mathcal{J}[y(x)] = 2\pi \int_{x_2}^{x_1} y \sqrt{1 + (y')} \, dx$. **12**

(5 × 12 = 60 Marks)

(Pages : 2) **P – 5247**

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, September 2022

Mathematics

MM 221 — ABSTRACT ALGEBRA

(2017 – 2019 Admission)

Time : 3 Hours Max. Marks : 75

Instructions : Answer **five** questions choosing Part – A or Part – B from each question and all questions carry equal marks.

1. (A) (a) Let G and H be finite cyclic groups. Show that $G \oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime.

(b) State and prove Cauchy's theorem for abelian groups. **5 + 10**

OR

(B) (a) Find five subgroups of S_5 of order 24.

(b) Show that every group of order p^2, where p is a prime, is isomorphic to Z_p or $Z_p \oplus Z_p$. **5 + 10**

2. (A) (a) State and prove Sylow's first theorem.

(b) Write down the Greedy algorithm for constructing an abelian group of order p^n. **10 + 5**

OR

(B) (a) Let G be an abelian group of prime power order and let a be an element of maximum order in G. Show that G can be written in the form $\langle a \rangle \times K$, where $K = \{x \in G / x^m = e\}$.

(b) Show that the only group of order 255 is Z_{255}. **10 + 5**

P.T.O.

3. (A) (a) State and prove the theorem for existence of factor rings.

(b) State and prove Gauss's lemma. **10 + 5**

OR

(B) (a) Let R be a commutative ring with unity and A be an ideal of R. Show that R/A is a field if and only if A is maximal.

(b) Let R be a ring with unity 1. Show that the mapping $\phi : Z \to R$ given by $n \to n.1$ is a ring homomorphism.

(c) Let $f(x) \in Z[x]$. Prove that if $f(x)$ is reducible over Q, then it is reducible over Z. **5 + 5 + 5**

4. (A) (a) Prove that every principal ideal domain is a unique factorization domain.

(b) Show that every finite field is perfect. **10 + 5**

OR

(B) (a) State and prove Kronecker's theorem.

(b) Show that every euclidean domain is a principal ideal domain. **10 + 5**

5. (A) (a) Let K be a finite extension field of the field E and let E be a finite extension field of the field F. Show that K is a finite extension field of F and $[K : F] = [K : E]\ [E : F]$.

(b) Show that a factor group of a solvable group is solvable. **10 + 5**

OR

(B) (a) Let F be a field of characteristic 0 and let $a \in F$. If E is splitting field of $x^n - a$ over F, show that the Galois group Gal(E/F) is solvable.

(b) If K is an algebraic extension of E and E is an algebraic extension of F, show that K is an algebraic extension of F. **10 + 5**

($5 \times 15 = 75$ Marks)

(Pages : 3) **P – 5248**

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, September 2022

Mathematics

MM 222 : REAL ANALYSIS II

(2017-2019 Admission)

Time : 3 Hours Max. Marks : 75

Instruction : Answer either Part A or Part B of each question. **All** questions carry equal marks.

UNIT I

I. (A) (a) Define Lebesgue outer measure and prove that it is countably sub -additive and translation invariant.

(b) Prove that every countable set has outer measure zero.

(c) Prove that the Lebesgue outer measure of an interval is its length.

3 + 2 + 10

(B) (a) Let $< E_n >$ be an infinite decreasing sequence of measurable sets. Let mE_1 be finite. Prove that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} mE_n$.

(b) Prove that the collection of measurable sets is a σ — algebra.

(c) Prove that there exists a non-measurable set.

5 + 5 + 5

P.T.O.

UNIT II

II. (A) (a) Show that $\int_1^\infty \frac{dx}{x} = \infty$.

(b) State and prove Fatou's lemma. Hence state and prove Lebesgue's Monotone convergence theorem.

(c) Show by an example that strict inequality can occur in Fatou's lemma.

3 + 10 + 2

(B) (a) If $f(x) = |x|$, find the first four derivatives at $x = 0$.

(b) Let f be a bounded function defined on the finite interval $[a,b]$. Prove that f is Riemann integrable over if $[a,b]$ iff f is continuous a.e.

(c) Let $[a,b]$ be a finite interval and let $f \in L(a,b)$ with indefinite integral *F.* Prove that $F' = f$ a.e. in $[a,b]$.

4 + 6 + 5

UNIT III

III. (A) (a) Show that if μ is a non-negative set function on a ring, is count-ably additive and is finite on some set, then μ is a measure.

(b) Prove that if μ is a a σ- finite measure on a ring ℝ, then it has a unique extension to the σ- ring 𝕊 (ℝ)

7 + 8

(B) (a) Prove that the class S^* of the μ^* measurable sets of *H(R)* is a σ— ring.

(b) If μ is a measure on a σ— ring 𝕊, then prove that the class $\overline{\mathbb{S}}$ of sets of the form $E\Delta N$ for any sets E, *N* such that E ∈ 𝕊 while *N* is contained in some set in 𝕊 of zero measure, is a σ— ring and the set function $\overline{\mu}$ defined by $\overline{\mu}(E\Delta N) = \mu(E)$ is a complete measure on $\overline{\mathbb{S}}$.

7 + 8

UNIT IV

IV. (A) (a) Prove that if $f, g \in L^p(\mu)$ and a, *b* are constants then $af + bg \in L^p(\mu)$.

(b) State and prove Holder's inequality. Also discuss when equality occurs in case when *f* and g are non-negative measurable functions.

5 + 10

(B) (a) State and prove Jensen's inequality.

(b) Prove that for $p \geq 1, L^p(\mu)$ is a complete metric space.

6 + 9

UNIT V

V. (A) (a) Prove that if f_n is a sequence of measurable functions which is fundamental in measure, then there exists a measurable function *f* such that $f_n \to f$ in measure.

(b) State and prove the Jordan decomposition theorem.

6 + 9

(B) State and prove the Radon- Nikodym theorem. **15**

(5 $\times$ 15 = 75 Marks)

(Pages : 3) **P – 5249**

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, September 2022

Mathematics

MM 223 : TOPOLOGY II

(2017-2019 Admission)

Time : 3 Hours Max. Marks : 75

Instruction : Answer either Part A or Part B of the equation

All questions carry equal marks.

UNIT I

I. (A) (a) Prove that the projection maps $p_i : X - X_i$ from a product space $X = X_1 \times X_2 \times ... \times X_n$ to the coordinate spaces are continuous.

(b) Prove that the product of a finite number of compact spaces is compact.

(c) Describe the weak topology for $\mathbb{R}$ generated by the family of constant functions $f : \mathbb{R} \to \mathbb{R}$. **5 + 7 + 3**

(B) (a) Let *X* be a space, *Y* be a set and let $f : X \to Y$ be a function from *X* onto *Y*. Define the quotient topology determined by *f*.

(b) Let *X* and *Y* be spaces and let $f : X \to Y$ be a continuous function from *X* onto *Y*. Prove that the function $h : X / \tilde{f} \to Y$ defined by $h([x]) = f(x), x \in X$ is a homeomorphism if and only if *Y* has the quotient topology determined by *f*.

(c) Show that every manifold is locally compact. **5 + 5 + 5**

P.T.O.

UNIT II

II. (A) (a) Define a Urysohn space. Prove that each Urysohn space is a Hausdorff space.

(b) Prove that the product of any family of regular spaces is regular.

(c) If X is a separable normal space and E a subset of X with $card \geq card\ \mathbb{R}$, then prove that E has a limit point in X. **5 + 5 + 5**

(B) State and prove Urysohn's lemma. **15**

UNIT III

III. (A) (a) Let A be a subset of a topological space X. Prove that for $x \in X, x \in \overline{A}$ if and only if there exists a filter on X which contains A and converges to x.

(b) Prove that X is a T_2-space if and only if each filter converges to at most one point.

(c) Let u be an ultrafilter on X and $A \subset X$ be such that $U \cap A \neq \phi$ for all $u \in u$. Prove that $A \in u$. **5 + 5 + 5**

(B) State and prove Tychonoff theorem. **15**

UNIT IV

IV. (A) (a) Prove that an interval [*a, b*] on the real line is contractible to *a*.

(b) With usual notations prove that if X is a space and x_0 a point of X, then $\Pi_1(X, x_0)$ is a group under the operation. **7 + 8**

(B) (a) State and prove the covering path property.

(b) Prove that the fundamental group $\prod_1(S^1)$ is isomorphic to the additive group $\mathbb{Z}$ of integers. **7 + 8**

UNIT V

V. (A) (a) Determine the fundamental group of a closed cylinder.

(b) Prove that if *D* is a deformation retract of a space *X* and x_0 is a point of *D*, then $\prod_1(X, x_0)$ and $\prod_1(D, x_0)$ are isomorphic. **7 + 8**

(B) (a) Let *X* be a space. Prove that every deformation retract of *X* is also a retract of *X.*

(b) Proving all the necessary results, state and prove the Brouwer fixed point theorem. **7 + 8**

(5 $\times$ 15 = 75 Marks)

(Pages : 3) **P – 5250**

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, September 2022

Mathematics

MM 224 : SCIENTIFIC PROGRAMMING WITH PYTHON

(2017 – 2019 Admission)

Time : 3 Hours Max. Marks : 50

Answer either Part A and Part B only of each question.

Each question carries **10** marks.

I. (A) (a) Write a python program to convert Fahrenheit to Celsius $(f = 9/5c + 32)$. **4**

(b) Write a program to modify the list $[1, 2, 3, 4]$ to make it $[1, 2, 3, 8]$. **3**

(c) What is the difference between Python's Module, Package and Library? **3**

OR

(B) (a) What are Logical Operators in Python? **4**

(b) Write a program to display even numbers within a range. Also display their sum and average. **3**

(c) Write a program to display the factorial of numbers from 1 to 20. **3**

P.T.O.

II. (A) (a) Use matplotlib.pyplot.plot to produce a plot of the functions $f(x) = e^{-x/10}\sin(\pi x)$ and $g(x) = xe^{-x/3}$ over the interval $[0, 10]$. Include labels for the x-and y-axes and a legend explaining which line is which plot. **4**

(b) What is tuple? What is the difference between list and tuple? **3**

(c) Write a Python program to plot $y = 2x^2 + 5x + 1$(for x from 0 to 1, 10 points), using pylab, with axes and title. Use red colored circles to mark the points. **3**

OR

(B) (a) What are the built-in functions that are used in Tuple? **4**

(b) Write Python code to plot $y = x^2$, with both the axes labeled. **3**

(c) Write a Python program to draw a bar chart. **3**

III. (A) (a) Evaluate the integral $\int_0^1 e^x \sin(x)\, dx$ using symbolic python. **4**

(b) Calculate the limit $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x}$ using sympy. **3**

(c) Solve the equation $x^3 + 1 = 0$ using SymPy's solve ( ) function. **3**

OR

(B) (a) How is symbolic Integration done in Python using SymPy? **4**

(b) Differentiate the functions $\sin(t)$, $\cos(t^2)$ using Sympy. **3**

(c) Explain the following functions in Python :

(i) plot( ), (ii) range ( ), (iii) arange, (iv) append ( ) **3**

IV. (A) (a) Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by method of False position. **5**

(b) Find a root of $f(x) = xe^x - 1 = 0$, using Bisection method, correct to three decimal places. **5**

OR

(B) (a) Explain Newton Raphson Method for solving equation of the form $f(x) = 0$. **5**

(b) Solve $x^3 - x - 1 = 0$ by Newton Raphson Method. **5**

V. (A) (a) Derive Newton's Cotes formula. Hence deduce Simpson's 3/8 rule. **7**

(b) Evaluate $\int_0^1 1 + x^2$ by using Simpson's 3/8 rule. **3**

OR

(B) (a) Explain Runge-Kutta method for solving an initial value problem : **4**

$$y' = f(x, y), \; y(x_0) = y_0$$

(b) Using the Runge-Kutta method of order 4, find $y(0.2)$ if

$dy/dx = (y - x)/(y + x)$, $y(0) = 1$ and $h = 0.2$. **6**

(5 × 10 = 50 Marks)